## Worked Example

## Use of the $\mathrm{CO}_{2}$ ventilation model

Consider an office $27 \mathrm{~m} \times 21 \mathrm{~m} \times 3 \mathrm{~m}(\mathrm{H})$ housing people at $9 \mathrm{~m}^{2} /$ person and a design ventilation rate of $10 \mathrm{~L} / \mathrm{s} /$ person. What is the $\mathrm{CO}_{2}$ concentration at different hours of the day assuming that office hour is 9 am to 6 pm with lunch time at 1 pm to 2 pm . The office is purged with fresh air after work.
$c=$ concentration of carbon dioxide in the room (ppm)
$Q^{\prime}=$ fresh air supply rate per occupant $\left(\mathrm{m}^{3} / \mathrm{s}\right)\left(10 \mathrm{~L} / \mathrm{s} /\right.$ person $=0.01 \mathrm{~m}^{3} / \mathrm{s} /$ person $)$
$V^{\prime}=$ occupied volume in the room per occupant $\left(\mathrm{m}^{3}\right)$
$t=$ time after room is occupied (sec)
$c_{a}=$ concentration of carbon dioxide in outdoor ambient air (take 380ppm)
$\dot{V}_{c}=$ volume of carbon dioxide produced per occupant (take $4 \times 10^{-6} \mathrm{~m}^{3} / \mathrm{s}$ )
$c_{0}=$ concentration of carbon dioxide in the room at time $t=0(\mathrm{ppm})$
$c=\left[\frac{10^{6} \dot{V}_{c}}{Q^{\prime}}+c_{a}\right]\left(1-e^{-\frac{Q^{\prime} t}{V^{\prime}}}\right)+c_{0} e^{-\frac{Q^{\prime} t}{V^{\prime}}}$

Volume of room $=27 \mathrm{~m} \times 21 \mathrm{~m} \times 3 \mathrm{~m}=1701 \mathrm{~m}^{3}$
The occupied volume per person $V^{\prime}=9 \mathrm{~m}^{2} \times 3 \mathrm{~m}=27 \mathrm{~m}^{3}$

Since the room is purges with fresh air overnight, therefore, it can be assumed that the $\mathrm{CO}_{2}$ concentration at the start of the working at 9am $=$ outdoor concentration $=380 \mathrm{ppm}$
Consider after one hour (i.e. at $10 \mathrm{am}, t=3600 \mathrm{~s}$ )
Substituting into the equation:

$$
\begin{aligned}
& \frac{Q^{\prime} t}{V^{\prime}}=\frac{0.01 \times 3600}{27}=1.33 \\
& e^{-\frac{Q^{\prime} t}{V^{\prime}}}=e^{-1.33}=0.264 \\
& c=\left[\frac{10^{6} \times 4 \times 10^{-6}}{0.01}+380\right](1-0.264)+380 \times 0.264=674 \mathrm{ppm}
\end{aligned}
$$

Using similar calculation, the $\mathrm{CO}_{2}$ concentration at different hours before lunch is given by:
10am = 674ppm
11am $=752 \mathrm{ppm}$
12noon $=773$ ppm
$1 \mathrm{pm}=778 \mathrm{ppm}$

At lunch hour, assume no one inside the office, but ventilation remains at constant rate
$c=\left[\frac{10^{6} \times 0}{0.01}+380\right](1-0.264)+778 \times 0.264=485 \mathrm{ppm}$
thus
at $2 \mathrm{pm}=485 \mathrm{ppm}$

The $\mathrm{CO}_{2}$ concentration for afternoon office hour is done similarly except putting the initial $\mathrm{CO}_{2}$ concentration at 485 ppm instead of 380 ppm in the morning.

The outdoor air change rate for the office
$=\frac{0.01 \times 3600}{27}=1.33 \mathrm{ACH}$

## Exercise

For energy saving purposes, the office in the worked example has reduced the ventilation rate such that a steady state $1000 \mathrm{ppm} \mathrm{CO}_{2}$ concentration can be achieved. What should be the ventilation rate (in $\mathrm{L} / \mathrm{s} /$ person or ACH )?

Try also to derive the equations in steps.
Solution
Using
$c=\left[\frac{10^{6} \dot{V}_{c}}{Q^{\prime}}+c_{a}\right]\left(1-e^{-\frac{Q^{\prime} t}{V^{\prime}}}\right)+c_{0} e^{-\frac{Q^{\prime} t}{V^{\prime}}}$ and inserting the necessary parameters
$1000=\left[\frac{10^{6} \times 4 \times 10^{-6}}{Q^{\prime}}+380\right]\left(1-e^{-n}\right)+380 e^{-n}$ where $n=\frac{Q^{\prime} t}{V^{\prime}}$
For steady state, $e^{-n} \rightarrow e^{-\infty} \rightarrow 0$
$1000=\left[\frac{4}{Q^{\prime}}+380\right]$
Thus
$Q^{\prime}=0.00645 \mathrm{~m}^{3} / \mathrm{s} /$ person $=6.5 \mathrm{~L} / \mathrm{s} /$ person
Air change rate
$=\frac{0.00645 \times 3600}{27}=0.86 \mathrm{ACH}$

