

Exercise 01 – Decision theory and decision tree

Question 1: Dr. No's Patient

Dr. No has a patient who is very sick. Without further treatment, this patient will die in about 3 months. The only treatment alternative is a risky operation. The patient is expected to live about 1 year if he survives the operation; however, the probability that the patient will not survive the operation is 0.3.

1. Draw a decision tree for this simple decision problem. Show all the probabilities and outcome values.
2. Let $U(x)$ denote the patient's utility function, where x is the number of months to live. Assuming that $U(12) = 1.0$ and $U(0) = 0$, how low can the patient's utility for living 3 months be and still have the operation be preferred? For the rest of the problem, assume that $U(3) = 0.8$.
3. Dr. No finds out that there is a less risky test procedure that will provide uncertain information that predicts whether or not the patient will survive the operation. When this test is positive, the probability that the patient will survive the operation is increased. The test has the following characteristics: (a) True-positive rate: The probability that the results of this test will be positive if the patient will survive the operation is 0.90; (b) False-positive rate: The probability that the results of this test will be positive if the patient will not survive the operation is 0.10. What is the patient's probability of surviving the operation if the test is positive?
4. Assuming the patient has the test done, at no cost, and the result is positive, should Dr. No perform the operation?
5. It turns out that the test may have some fatal complications, i.e., the patient may die during the test. Draw a decision tree showing all the options and consequences of Dr. No's problem.
6. Suppose that the probability of death during the test is 0.005 for the patient. Should Dr. No advise the patient to have the test prior to deciding on the operation?

Question 2: Product Decision

To absorb some short-term excess production capacity at its manufacturing plant, ABC Company is considering a short manufacturing run for either of two new products, a temperature sensor or a pressure sensor. The market for each product is known if the products can be successfully developed. However, there is some chance that it will not be possible to successfully develop them.

Revenue of \$1,000,000 would be realized from selling the temperature sensor and revenue of \$400,000 would be realized from selling the pressure sensor. Both of these amounts are net of production cost but do not include development cost. If development is unsuccessful for a product, then there will be no sales, and the development cost will be totally lost. Development cost would be \$100,000 for the temperature sensor and \$10,000 for the pressure sensor.

Which, if either, of these products should ABC Company attempt to develop? Draw a decision tree diagram to represent and analyse the situation. Assume the probability of development success is 0.5 for the temperature sensor and 0.8 for the pressure sensor.