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Decision theory and decision tree

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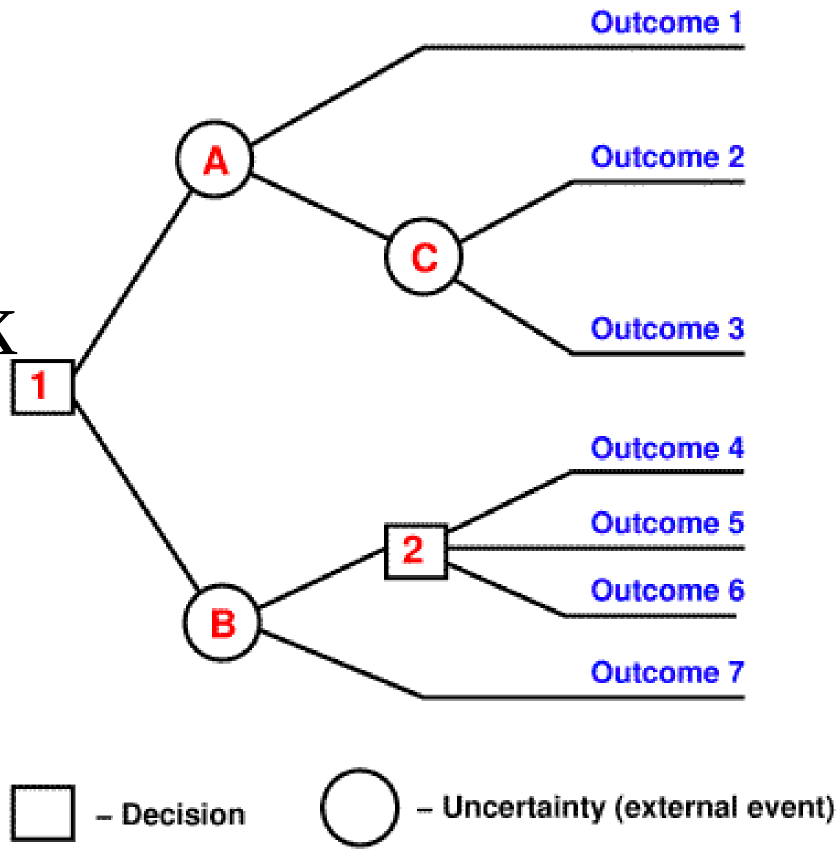


Decision Theory

- Decision theory (DT) represents a generalized approach to *decision making*. It enables the decision maker:
  - To analyze a set of complex situations with many alternatives and many different possible consequences
  - To identify a course of action consistent with the basic economic and psychological desires of the decision maker

Decision Making…

Decision Theory

- ***Decision making*** is an integral part of management planning, organizing, controlling and motivation processes
  - The decision maker selects one strategy (course of action) over others depending on some criteria, like utility, sales, cost or rate of return
  - Is used whenever an organization or an individual faces a problem of decision making or dissatisfied with the existing decisions or when alternative selection is specified

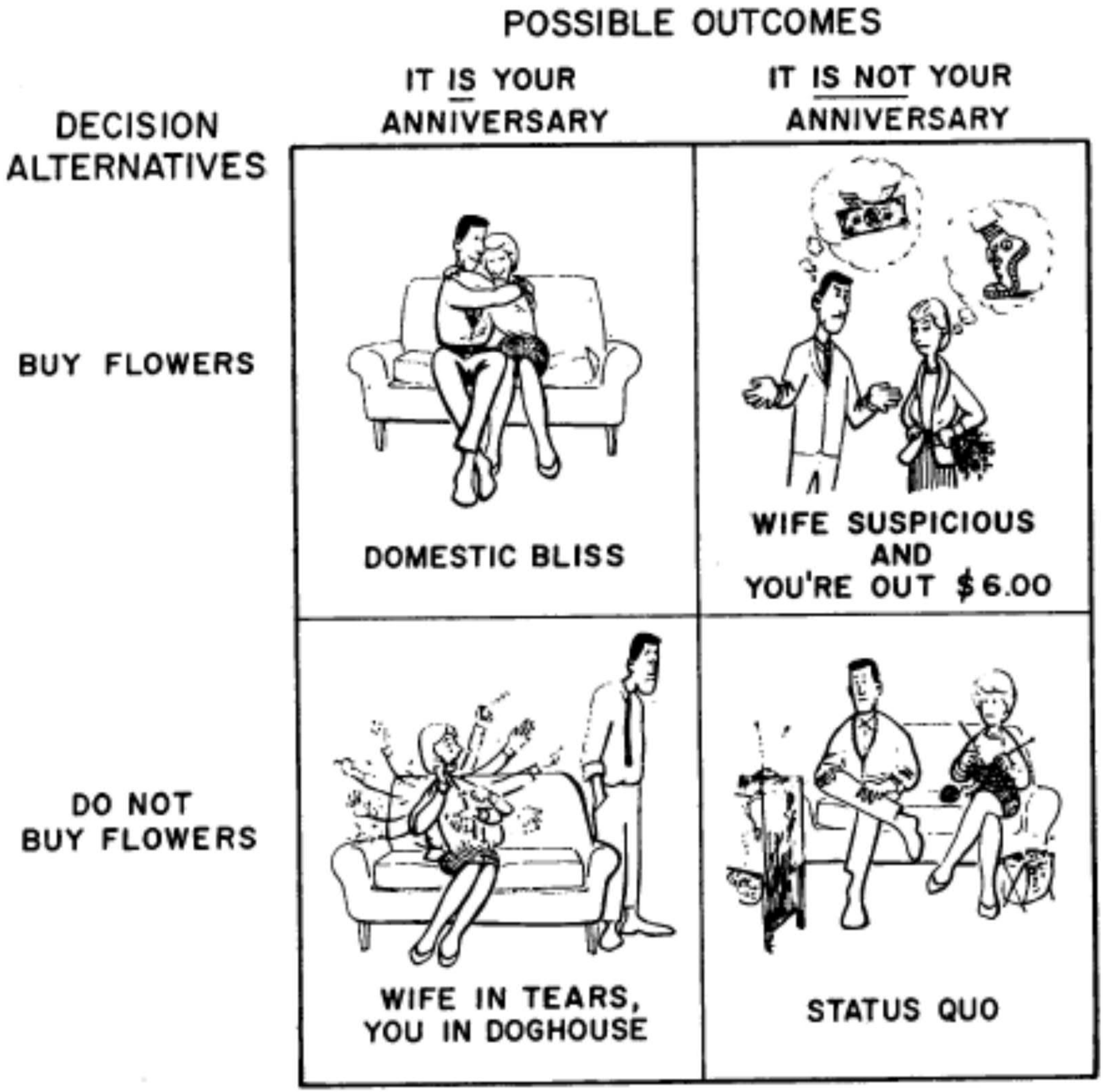
Decision Theory

- Types of decisions
  - Strategic Decision: concerned with external environment of the organization
  - Administrative Decision: concerned with structuring and acquisition of the organization’s resources so as to optimize the performance of the organization
  - Operating Decision: concerned with day to day operations of the organization such as pricing, production scheduling, inventory levels, etc.

Decision Theory

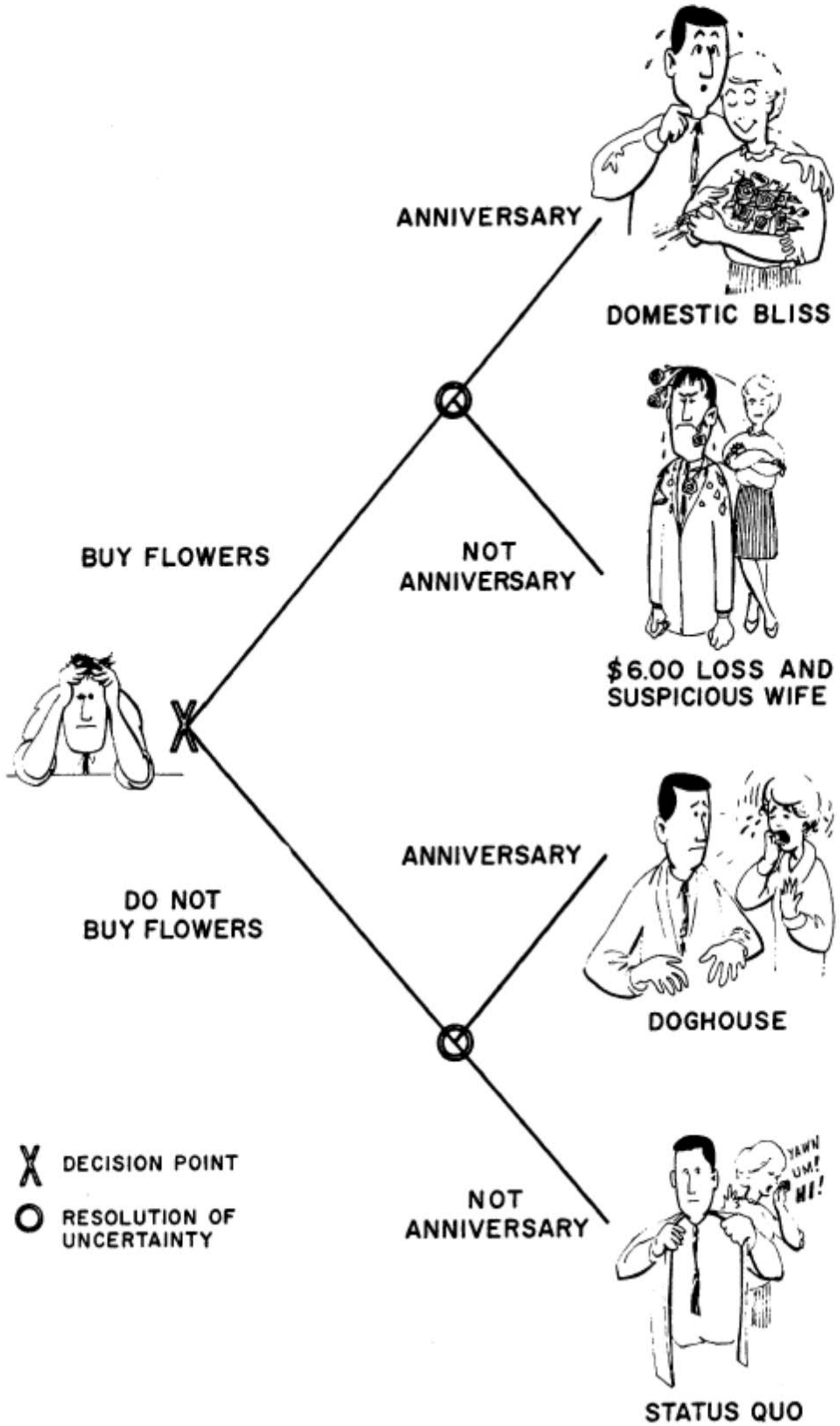
- Decision theory problems are characterized by the following:
  - A decision criterion
  - A list of alternatives
  - A list of possible future events (states of nature)
  - Payoffs associated with each combination of alternatives and events (e.g. a payoff matrix)
  - The degree of certainty of possible future events

Anniversary problem payoff matrix



(Source: North, D. W., 1968. A tutorial introduction to decision theory, *IEEE Trans. on Systems Science and Cybernetics*, vol. SSC-4, no. 3)

Diagram of anniversary decision



(Source: North, D. W., 1968. A tutorial introduction to decision theory, *IEEE Trans. on Systems Science and Cybernetics*, vol. SSC-4, no. 3)

Classification of Decision Problems

Point of View	**Types of Decision Problems**
possibility and complexity of algoritmization	ill-structured, well-structured, semi-structured
number of criteria	one criterion, multiple criteria
character of the decision maker	individual, group
number of decision making periods	one stage, multiple stage
relationships among decision makers	conflict, cooperative, non-cooperative
degree of certainty for future events	complete certainty, risk, uncertainty

Decision Theory

- Solution steps to any decision problem:
  - 1. Identify the problem
  - 2. Specify objectives and the decision criteria for choosing a solution
  - 3. Develop alternatives
  - 4. Analyze and compare alternatives
  - 5. Select the best alternative
  - 6. Implement the chosen alternative
  - 7. Verify that desired results are achieved

Decision Theory

- Elements related to all decisions
  - Goals to be achieved: objectives which the decision maker wants to achieve by his actions
  - The decision maker: refers to an individual or an organization
  - Courses of action (A_i): also called "Action" or "Decision Alternatives". They are under the control of decision maker
  - States of nature (ϕ_j): Exhaustive list of possible future events. Decision maker has no direct control over the occurrence of particular event

Decision Theory

- Elements related to all decisions (cont'd)
  - The preference or value system: criteria that the decision maker uses in making a choice of the best course of action
  - Payoff: effectiveness associated with specified combination of a course of action and state of nature. Also known as profits or conditional values
  - Payoff table: a summary table of the payoffs
  - Opportunity loss table: incurred due to failure of not adopting most favorable course of action or strategy. Found separately for each state of nature

Payoff table (or decision matrix)

States of Nature O_j	Courses of Action A_i			
	A_1	A_2	…	A_m
Φ_1	$v(A_1, \Phi_1)$	$v(A_2, \Phi_1)$	…	$v(A_m, \Phi_1)$
Φ_2	$v(A_1, \Phi_2)$	$v(A_2, \Phi_2)$	…	$v(A_m, \Phi_2)$
…	…	…	…	…
Φ_n	$v(A_1, \Phi_n)$	$v(A_2, \Phi_n)$	…	$v(A_m, \Phi_n)$

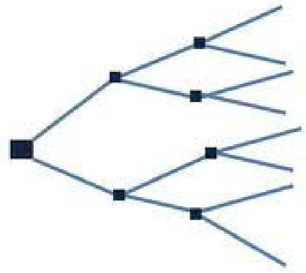
A_i = alternate course of action i; $i = 1, 2, \ldots, m$
Φ_j = state of nature j; $j = 1, 2, \ldots, n$
v = payoff or value associated with specified combination of a course of action and state of nature

Decision Theory

- Types of environment:
  - 1. Decision making under certainty
    - Future "states of nature" are known
    - Will choose the alternative that has the highest payoff (or the smallest loss)
  - 2. Decision making under uncertainty
    - Future "states of nature" are uncertain
    - Depends on the degree of decision maker's optimism
  - 3. Decision making under risk
  - 4. Decision making under conflict (Game Theory)

Decision theory applies

Decision Tree

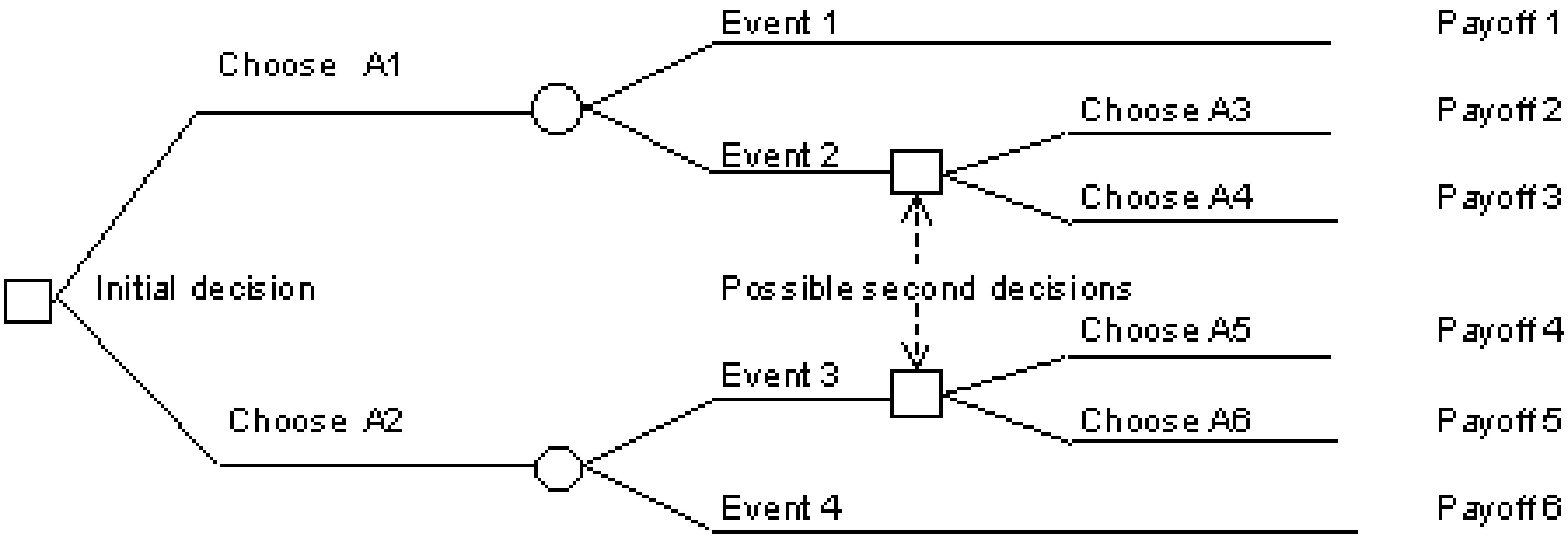


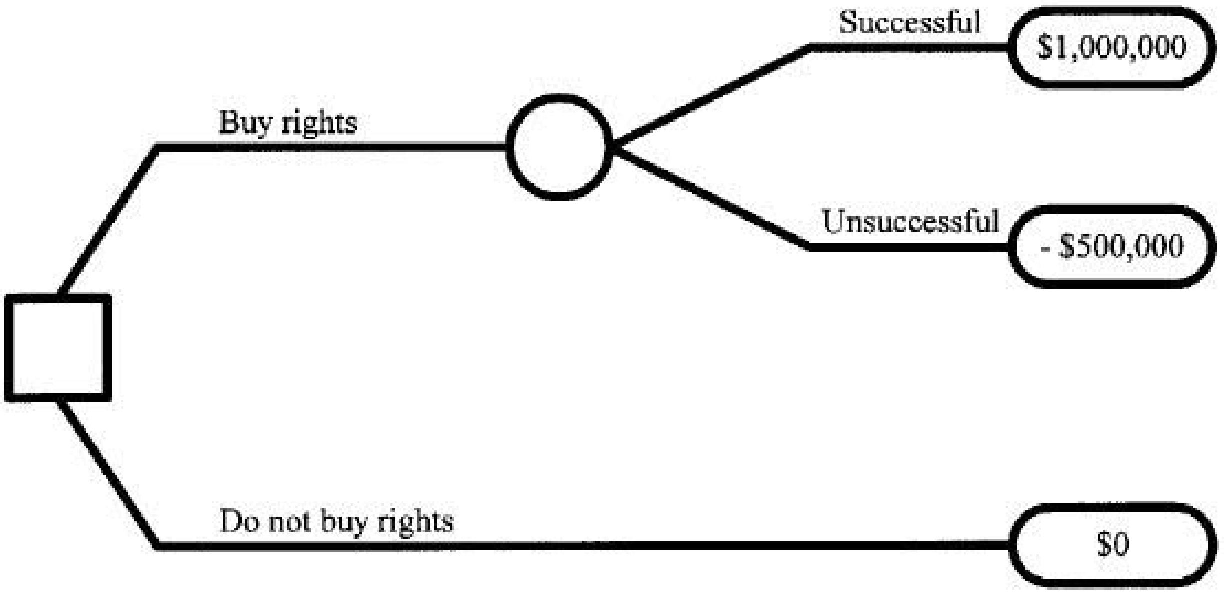
- It is a decision support tool that uses a tree-like graph or model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility
- Commonly used in operations research, specifically in decision analysis, to help identify a strategy most likely to reach a goal. Another use is as a descriptive means for calculating conditional probabilities
- It enables people to decompose a large complex decision problem into several smaller problems

Decision Tree

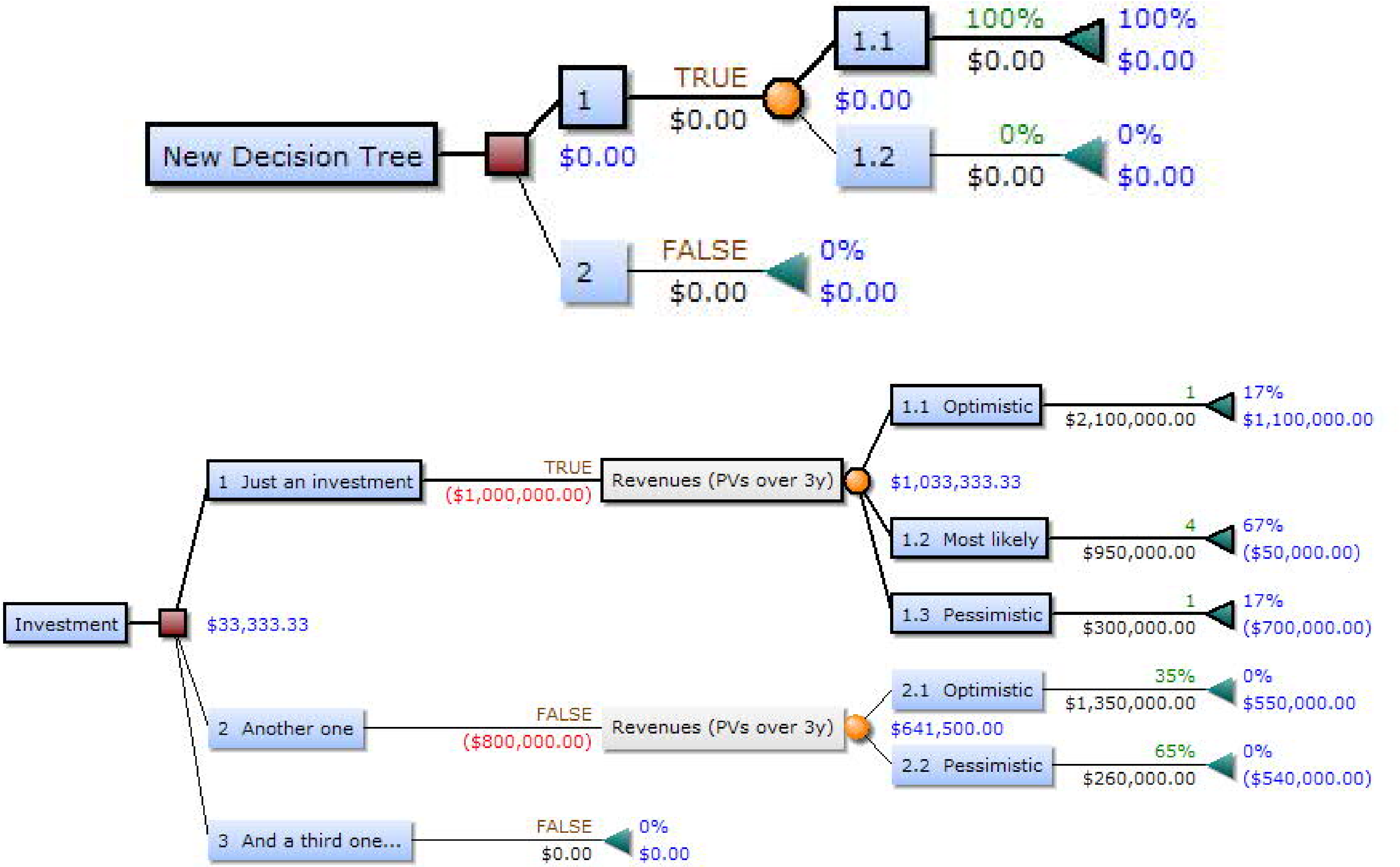
- A decision tree consists of 3 types of nodes:-
  - 1. Decision nodes - commonly represented by squares
  - 2. Chance nodes - represented by circles
  - 3. End nodes - represented by triangles/ellipses
- A decision tree has only burst nodes (splitting paths) but no sink nodes (converging paths)

Examples of decision trees





Examples of decision trees



(Source: http://en.wikipedia.org/wiki/Decision_tree)

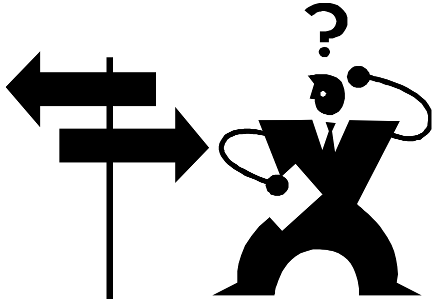
Decision Tree

- Advantages of decision trees:
  - Are simple to understand and interpret
  - Have value even with little hard data
  - Possible scenarios can be added
  - Worst, best and expected values can be determined for different scenarios
  - Use a white box model. If a given result is provided by a model
  - Can be combined with other decision techniques e.g. Net Present Value calculations

Decision Tree

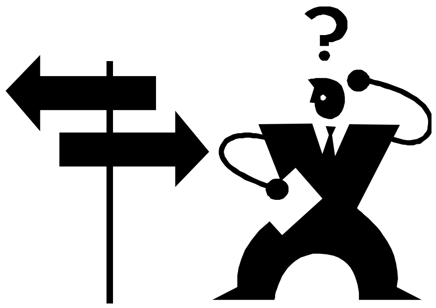
- Disadvantages of decision trees:
  - For data including categorical variables with different number of levels, information gain in decision trees are biased in favor of those attributes with more levels
  - Calculations can get very complex particularly if many values are uncertain and/or if many outcomes are linked

Decision Criteria



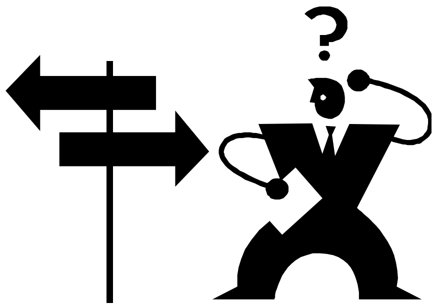
- Decision making under uncertainty
- Typical decision rules with no knowledge of future probabilities:
  - 1. Criterion of Optimism or MaxiMax
  - 2. Criterion of Pessimism or MaxiMin
  - 3. Hurwicz Criterion
  - 4. MiniMax Regret Criterion (Regret payoff)
  - 5. Laplace Criterion (Equally Likelihood)

Decision Criteria



- The *MaxiMax* rule:
  - It is appropriate for extreme optimists that expect the most favourable situation (they choose the alternative that could result in the max. payoff)
  - Under this rule, the decision maker will find the largest payoff in the decision matrix and select the alternative associated with it (the largest payoff is determined for each alternative and then the largest payoff from these values is selected; therefore “maximax”) $\max_{A_i}\left[\max_{\emptyset_j}\{v(A_i, \emptyset_j)\}\right]$

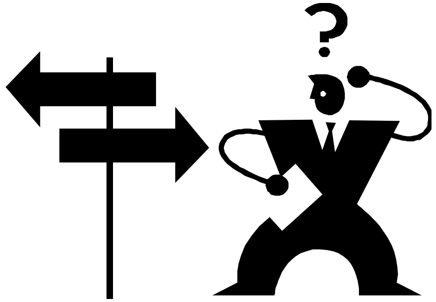
Decision Criteria



- The *MaxiMin* rule (Wald criterion):
  - It represents a pessimistic approach when the worst decision results are expected (an extremely conservative type of decision rule)
  - The decision maker determines the smallest payoff for each alternative and then chooses the alternative that has the best (maximum) of the worst (minimum) payoffs (therefore “maximin”)

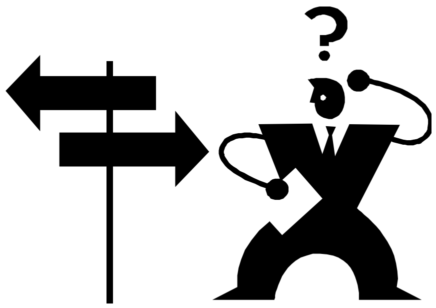
$$\max_{A_i}\left[\min_{\emptyset_j}\{v(A_i, \emptyset_j)\}\right]$$

Decision Criteria



- Example: A food product company is contemplating introduction of a revolutionary new product with new packaging to replace the existing product at much price (S1) or a moderate change in the composition of the existing product with a new packaging at a small increase in price (S2) or a small change in the composition of the existing except the word 'new' with a negligible increase in price (S3). The three possible states of nature of events are:
  - (i) High increase in sales (N1)
  - (ii) No change in sales (N2)
  - (iii) Decreases in sales (N3)
- Its marketing department worked out the payoffs in terms of yearly net profit for each course of action for these events (expected sales)

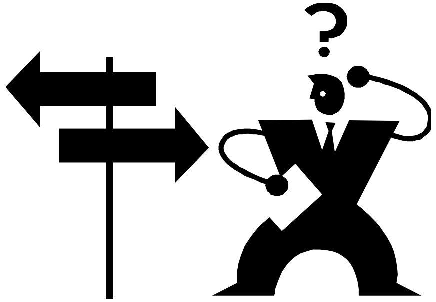
Decision Criteria



- Example (cont'd): summary of the results:

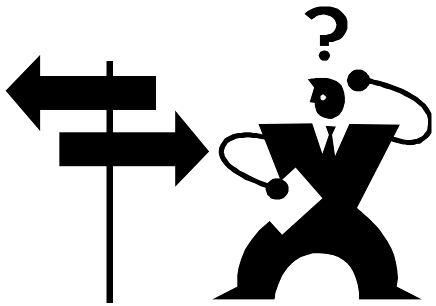
States of Nature Oj	Courses of Action		
	S1	S2	S3
N1 (high increase)	700,000	500,000	300,000
N2 (no change)	300,000	450,000	300,000
N3 (decrease)	150,000	0	300,000
MaxiMin Payoff	150,000	0	300,000
MaxiMax Payoff	700,000	500,000	300,000

Decision Criteria



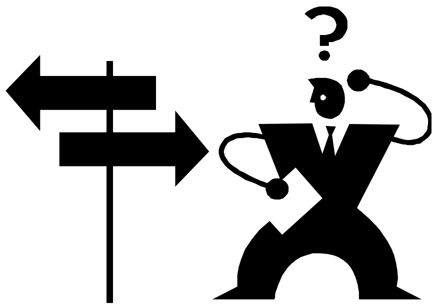
- The *Hurwicz α-criterion* (criterion of realism)
  - It represents a compromise between the optimistic and the pessimistic approach
    - The measure of optimism and pessimism is expressed by an *optimism-pessimism index α* (0,1) . The more this index is near to 1, the more the decision maker is optimist. By means of the index , a weighted average of the best payoff (its weight = α) and the worst payoff (its weight = 1 - α) is computed for each alternative and the alternative with the largest weighted average should be chosen. If α =1, the above rule is the maximax criterion, whereas if α =0, it is the maximin rule

Decision Criteria



- The *MiniMax regret rule* (Savage criterion)
  - Take all payoffs into account and represent a pessimistic approach used for an opportunity loss table
  - The opportunity loss reflects the difference between each payoff and the best possible payoff in a column (it can be defined as the amount of profit foregone by not choosing the best alternative for each state of nature). Hence, opportunity loss amounts are found by identifying the greatest payoff in a column and, then, subtracting each of the other values in the column from that payoff. The values in an opportunity loss table can be viewed as potential "regrets" that might be suffered as the result of choosing various alternatives. Minimizing the maximum possible regret requires identifying the maximum opportunity loss in each row and, then, choosing the alternative that would yield the minimum of those regrets (this alternative has the "best worst")

Decision Criteria



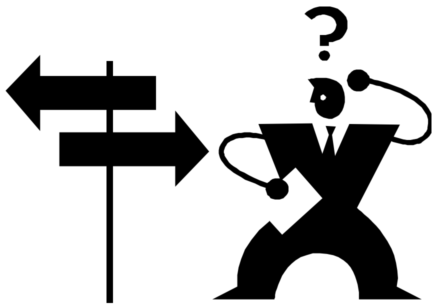
- The *MiniMax regret rule* (cont'd)
  - It assumes that a new loss matrix is constructed as follows. Then apply the MiniMax criterion

$$r(A_i, \emptyset_j) = \begin{cases} \max\limits_{A_k}\{v(A_k, \emptyset_j)\} - v(A_i, \emptyset_j), & \textit{if } v \textit{ is profit} \\ v(A_i, \emptyset_j) - \min\limits_{A_k}\{v(A_k, \emptyset_j)\}, & \textit{if } v \textit{ is loss} \end{cases}$$

$$\min_{A_i}\left[\max_{\emptyset_j}\{r(A_i, \emptyset_j)\}\right]$$

  - The disadvantage of MiniMax regret criterion is the inability to factor row differences. It is removed in the further rule that incorporates more of information for the choice of the best alternative

Decision Criteria



- The *principle of insufficient reason* (Laplace criterion)
  - It assumes that all states of nature are equally likely, i.e. $P(\emptyset = \emptyset_j) = \frac{1}{n} \quad \forall j$. Under this assumption, the decision maker can compute the average payoff for each row (the sum of the possible consequences of each alternative is divided by the number of states of nature) and, then, select the alternative A_i that has the highest row average

$$\max_{A_i} \left\{ \frac{1}{n} \sum_{j=1}^{n} v(A_i, \emptyset_j) \right\}$$

Decision Analysis

- Example:
  - A developer must decide how large a luxury sports complex to build – small, medium, or large. The profitability of this complex depends upon the future level of demand for the complex
    - States of nature: the states of nature could be defined as low demand and high demand
    - Alternatives: could decide to build a small, medium, or large complex
    - Payoffs: the profit for each alternative under each potential state of nature is going to be determined

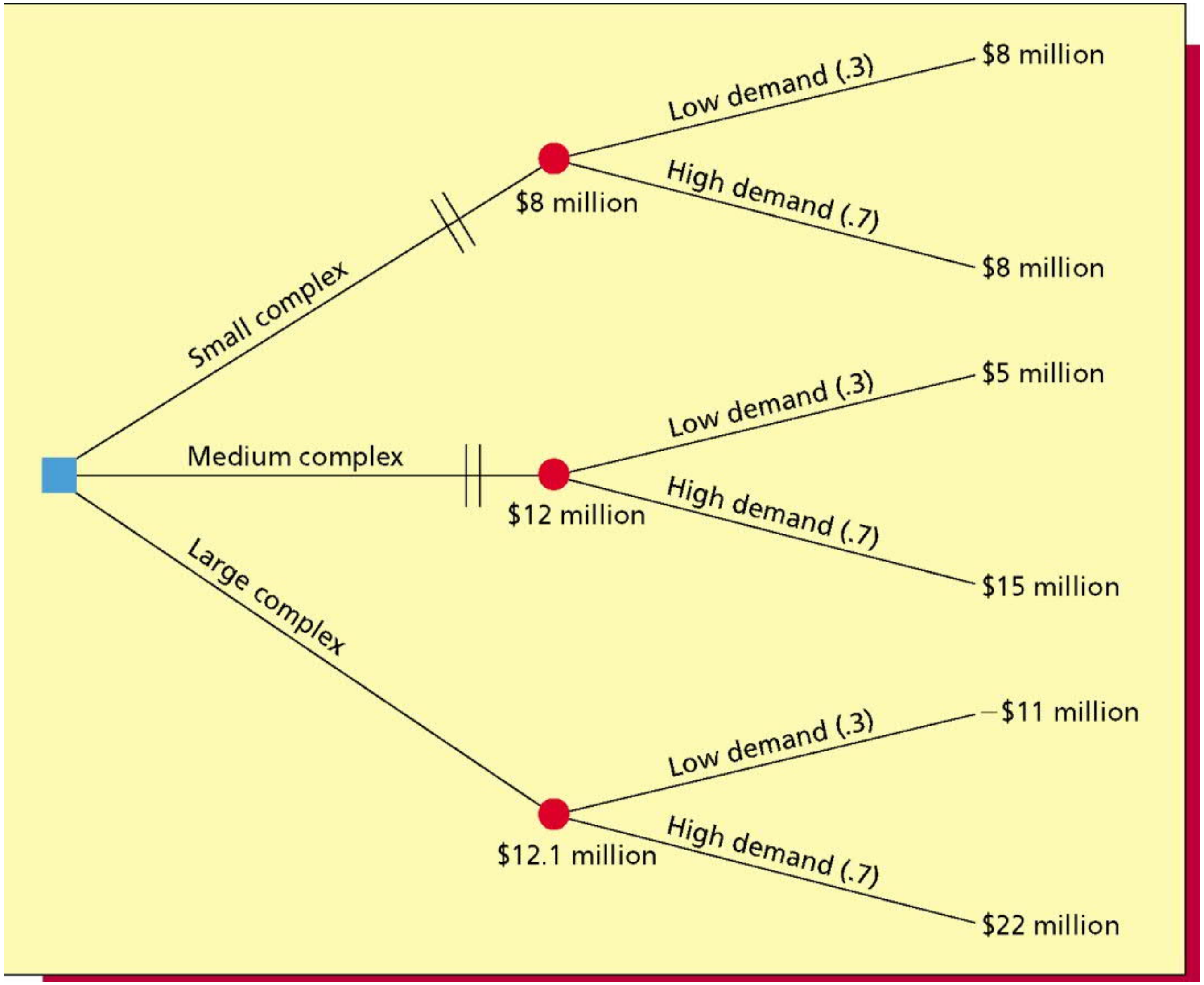
Example of decision analysis

Payoff Table

	States of Nature	
Alternatives	**Low**	**High**
Small	8	8
Medium	5	15
Large	-11	22

(payoffs in millions of dollars)

Decision Tree



Decision Tree Legend

Branch

Decision point (node)

State of nature (node)

Decision Analysis

- Decision making without probabilities: Three commonly used criteria for decision making when probability information regarding the likelihood of the states of nature is unavailable are:
  - Optimistic (maximax) approach
  - Conservative (maximin) approach
  - Minimax regret approach

Decision Analysis

- Optimistic (maximax) approach:
  - The optimistic approach would be used by an optimistic decision maker
  - The <u>decision with the best possible payoff</u> is chosen
  - If the payoff table was in terms of costs, the <u>decision with the lowest cost</u> would be chosen
  - If the payoff table was in terms of profits, the <u>decision with the highest profit</u> would be chosen

Decision Analysis

- Conservative (maximin) approach
  - Used by a conservative decision maker
    - For each decision the worst payoff is listed and then the decision corresponding to the best of these worst payoffs is selected (Hence, the <u>worst possible payoff is maximized</u>)
    - If the payoff was in terms of costs, the maximum costs would be determined for each decision and then the decision corresponding to the minimum of these maximum costs is selected. (Hence, the <u>maximum possible cost is minimized</u>)
    - If the payoff was in terms of profits, the minimum profits would be determined for each decision and then the decision corresponding to the maximum of these minimum profits is selected. (Hence, the <u>minimum possible profit is maximized</u>)

Decision Analysis

- Minimax regret approach
  - Requires the construction of a regret table or an opportunity loss table. This is done by calculating for each state of nature the difference between each payoff and the best payoff for that state of nature
  - Then, using this regret table, the maximum regret for each possible decision is listed
  - The decision chosen is the one corresponding to the minimum of the maximum regrets

Example of decision analysis: sports complex (cont'd)

Decision making under uncertainty:

Maximin: Identify the minimum (or worst) possible payoff for each alternative and select the alternative that maximizes the worst possible payoff. Pessimistic.

Example:

	States of Nature		
Alternatives	**Low**	**High**	**Min**
Small	8	8	8
Medium	5	15	5
Large	-11	22	-11
	(payoffs in millions of dollars)		

Maximin decision

Maximin payoff

Maximax: Identify the maximum (or best) possible payoff for each alternative and select the alternative that maximizes the best possible payoff. Optimistic.

Example:

	States of Nature		
Alternatives	**Low**	**High**	**Max**
Small	8	8	8
Medium	5	15	15
Large	-11	22	22
	(payoffs in millions of dollars)		

Maximax decision

Maximax payoff

Example of decision analysis: sports complex (cont'd)

If the minimax regret approach is selected:

Step 1: Determine the best payoff for each state of nature and create a regret table.

	STATES OF NATURE	
Alternatives	**Low**	**High**
Small	8	8
Medium	5	15
Large	-11	22
	Best Profit for Low 8	Best Profit for High 22

Example of decision analysis: sports complex (cont'd)

If the minimax regret approach is selected:

Step 1: Create a regret table (continued).

	STATES OF NATURE	
Alternatives	**Low**	**High**
Small	0	14
Medium	3	7
Large	19	0

For a profit payoff table, entries in the regret table represent profits that could have been earned.

If they knew in advanced that the demand would be low, they would have built a small complex. Without this "psychic insight", if they decided to build a medium facility and the demand turned out to be low, they would regret building a medium complex because they only made 5 million dollars instead of 8 million had they built a small facility instead. They regret their decision by 3 million dollars.

Example of decision analysis: sports complex (cont'd)

If the minimax regret approach is selected:

Step 2: Create a regret table (continued).

Step 3: Determine the maximum regret for each decision.

	STATES OF NATURE		
Alternatives	**Low**	**High**	**Max Regret**
Small	0	14	14
Medium	3	7	7
Large	19	0	19

Regret not getting a profit of 19 more than not making a profit of 0.

Example of decision analysis: sports complex (cont'd)

If the minimax regret approach is selected:

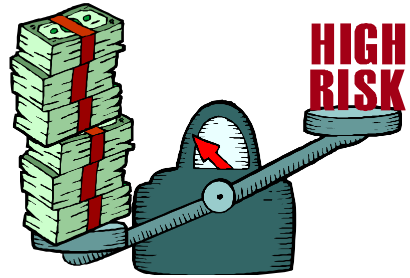
Step 4: Select the decision with the minimum value from the column of max regrets.

	STATES OF NATURE		
Alternatives	**Low**	**High**	**Max Regret**
Small	0	14	14
Medium	3	7	7
Large	19	0	19

Minimax Regret decision

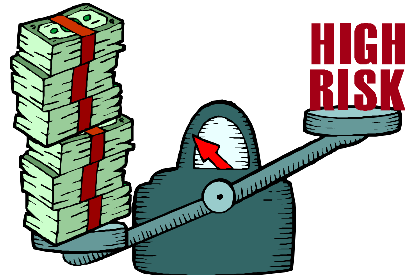
Minimax Regret payoff

Decision Making under Risk



- The decision maker does not know which state of nature will occur but can estimate the *probability* of occurrence for each state
- These probabilities may be subjective (they usually represent estimates from experts in a particular field), or they may reflect historical frequencies

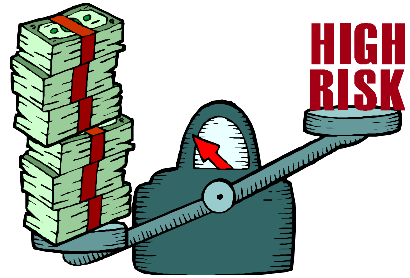
Decision Making under Risk



- An interesting example:
  - “Deal or no deal?”
  - TV game shows
  - Losses *vs* Gains
  - Risk-averse *vs* risk-seeking
  - A natural decision-making experiment

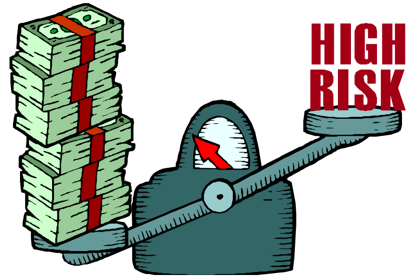
- Future probabilities estimated or known:
  - Expected Value (EV)
  - Expected Opportunity Loss (EOL)

Decision Making under Risk



- Decision making with probabilities:
  - If probabilistic information regarding the states of nature is available, one may use the expected value (EV) approach
  - Here the expected return for each decision is calculated by summing the products of the payoff under each state of nature and the probability of the respective state of nature occurring
  - The decision yielding the best expected return is chosen

Decision Making under Risk

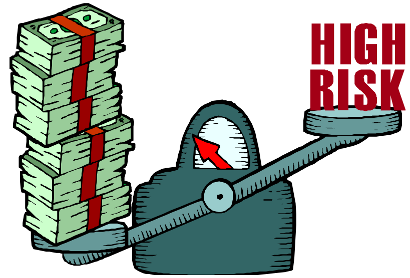


- Decision making with probabilities: (cont'd)
  - The expected value of a decision alternative is the sum of weighted payoffs for the decision alternative
  - The expected value (EV) of decision alternative d_i is defined as:
  - $$\mathrm{EV}(d_i) = \sum_{j=1}^{N} P(s_j) V_{ij}$$
    - where: N = the number of states of nature

      $P(s_j)$ = the probability of state of nature s_j

      V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

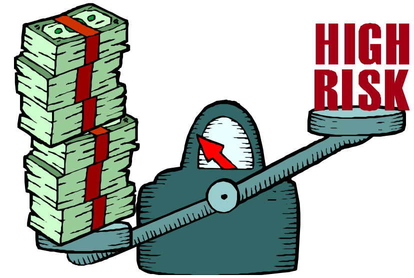
Decision Making under Risk



- Expected Monetary Value (EMV)
  - EMV for the specified course of action is the weighted average payoff, i.e. the sum of the product of the payoff for the several combination of courses of action and states of nature multiplied by the probability of occurrence of each outcome

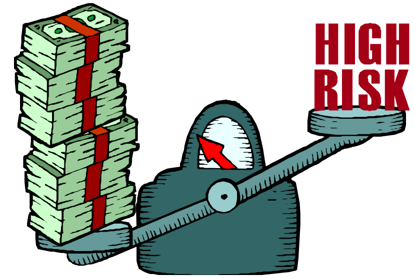
$$EMV = \sum_{n=1}^{N} Value_n \times Probability_n$$

Decision Making under Risk



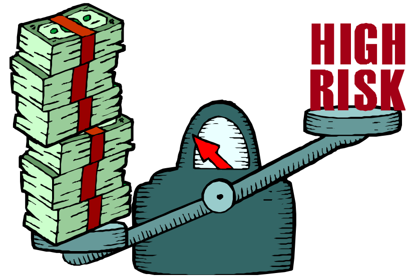
- Expected Profit with Perfect Information (EPPI)
  - EPPI is the maximum attainable expected monetary value (EMV) based on perfect information about the state of nature that will occur
  - EPPI may be defined as the sum of the product of best state of nature corresponding to each optimal course of action and its probability

Decision Making under Risk



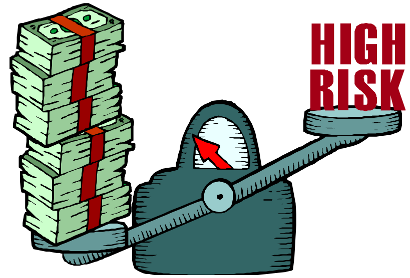
- Expected Value of Perfect Information (EVPI)
  - EVPI is defined as the maximum amount one would pay to obtain perfect information about the state of nature that would occur
  - EMV* represents the maximum attainable expected monetary value given only the prior outcome probabilities, with no information as to which state of nature will actually occur
  - (EVPI) = EPPI – EMV*

Decision Making under Risk



- Expected Opportunity Loss (EOL)
  - Another useful way of maximizing monetary value is to maximize the EOL or expected value of regret
  - The conditional opportunity loss (COL) for a particular course of action is determined by taking the difference between the payoff value of the most favourable course of action and some other course of action
  - Thus, opportunity loss can be obtained separately for each course of action by first obtaining the best state of nature for the prescribed course of action and then taking the difference between that best outcome and each outcome for those courses of action

Decision Making under Risk



- Sources of probabilities:
  - Sample Information - a study or research analysis of the environment is used to assess the probability or occurrence of the event
  - Historical Records - available from to files
  - Subjective Probabilistic - probability may be subjectively assessed based on judgment, sample information and historical records

Examples

- Example: calculate EMV, EPPI, EVPI, EOL
  - An electric manufacturing company has seen its business expanded to the point where it needs to increase production beyond its existing capacity. It has narrowed the alternatives to two approaches to increase the maximum production capacity:
    - (a) Expansion, at a cost of $8 million, or
    - (b) Modernization at a cost of $5 million
  - Both approaches would require the same amount of time for implementation

Examples

- Example: calculate EMV, EPPI, EVPI, EOL
  - Management believes that over the required payback period, demand will either be high or moderate. Since high demand is considered to be somewhat less likely than moderate demand, the probability of high demand has been setup at 0.35. If the demand is high, expansion would gross an estimated additional $12 million but modernization only an additional $6 million, due to lower maximum production capability. On the other hand, If the demand is moderate, the comparable figures would be $7 million for expansion and $5 million for modernization

Examples

- Calculating EMV, EPPI, EVPI, EOL: (cont'd)

States of Nature:	Courses of Action:
O1 = High Demand O2 = Moderate demand	S1 = Expand S2 = Modernize

States of Nature Oj	Probability P(Oj)	Courses of Action	
		S1 (Expand)	S2 (Modernize)
O1 = High Demand	0.35	12 - 8 = 4	6 - 5 = 1
O2 = Moderate demand	0.65	7 - 8 = -1	5 - 5 = 0

- Calculate EMV's for courses of action S1 and S2:
- EMV (S1) = (0.35) x (4) + (0.65) x (-1) = 1.40 – 0.65 = $ 0.75 million
- EMV (S2) = (0.35) x (1) + (0.65) x (0) = $ 0.35 million

Examples

- Calculating EMV, EPPI, EVPI, EOL: (cont'd)
  - To compute EVPI, we calculate EPPI
    - EVPI = EPPI – EMV* = 1.40 – 0.75 = $ 0.65 million

States of Nature Oj	Probability P(Oj)	Optimal Courses of Action	Conditional Profit	Weighted Profit
O1 = High Demand	0.35	S1	4	4 x 0.35 = 1.40
O2 = Moderate demand	0.65	S2	0	0 x 0.65 = 0
			EPPI =	1.40

Examples

- Calculating EMV, EPPI, EVPI, EOL: (cont’d)
  - Calculate EOL’s for courses of action S1 and S2:
    - EOL (S1) = (0.35) X (0) + (0.65) X (1) = **$ 0.65 million**
    - EOL (S2) = (0.35) X (3) + (0.65) X (0) = $ 1.05 million
  - Therefore, we select course of action S1 to produce the smallest expected opportunity loss

States of Nature Oj	Probability P(Oj)	Courses of Action		Loss ($ Million)	
		S1 (Expand)	S2 (Modernize)	S1 (Expand)	S2 (Modernize)
O1 = High Demand	0.35	4	1	0	3
O2 = Moderate demand	0.65	-1	0	1	0

Examples

- Example: decision making with probabilities
  - ABC Restaurant is contemplating opening a new restaurant on Main Street. It has three different models, each with a different seating capacity. They estimate that the average number of customers per hour will be 80, 100, or 120. The payoff table (profits) for the three models is developed

Example of decision analysis: ABC Restaurant
Decision making with probabilities

- **Payoff Table**

Average Number of Customers Per Hour

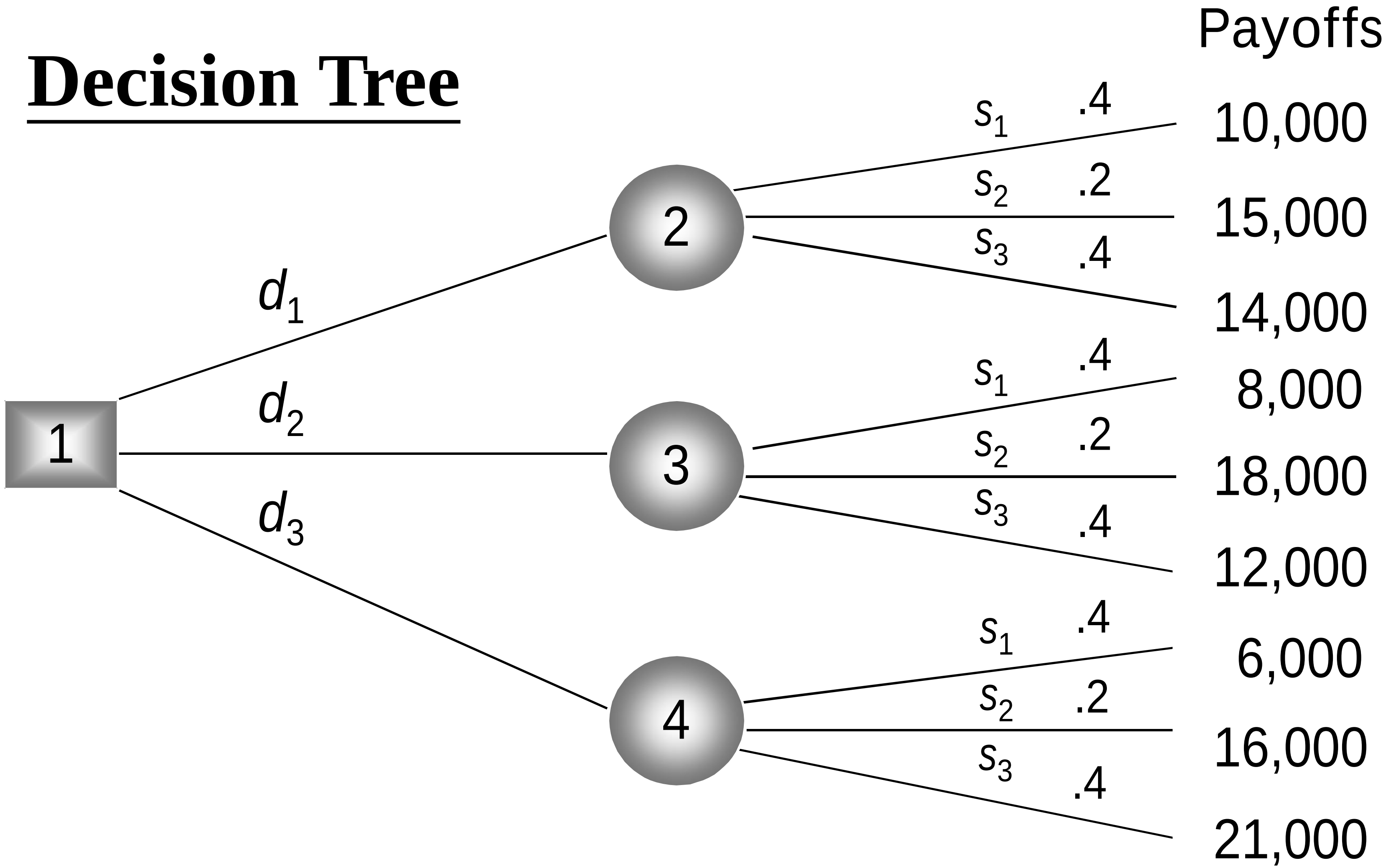
	$s_1 = 80$	$s_2 = 100$	$s_3 = 120$
Model A	\$10,000	\$15,000	\$14,000
Model B	\$ 8,000	\$18,000	\$12,000
Model C	\$ 6,000	\$16,000	\$21,000

- **Expected Value Approach**

Calculate the expected value for each decision. The decision tree on the next slide can assist in this calculation. Here d_1, d_2, d_3 represent the decision alternatives of models A, B, C, and s_1, s_2, s_3 represent the states of nature of 80, 100, and 120.

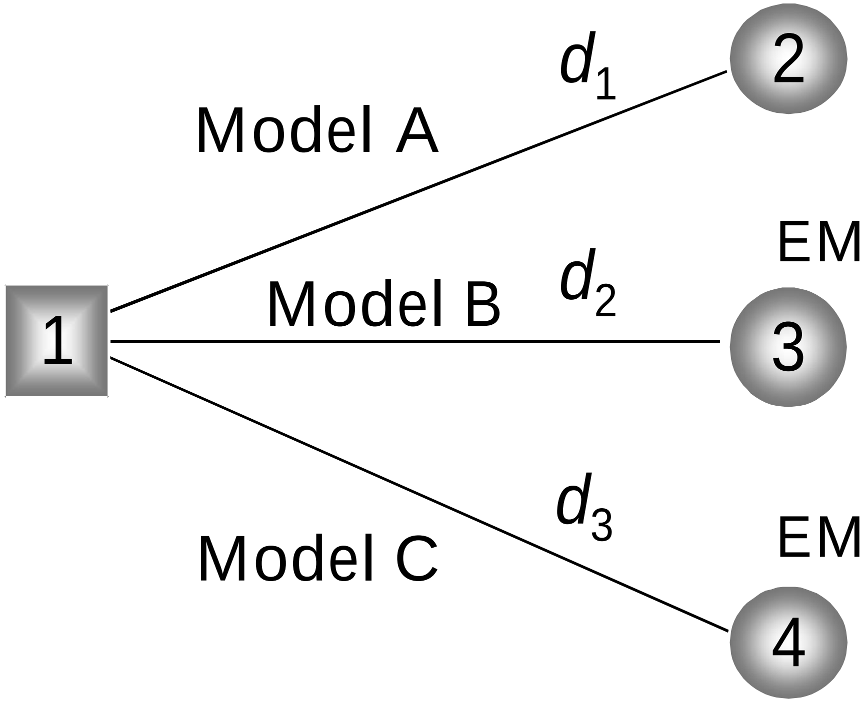
Example of decision analysis: ABC Restaurant (cont'd)
Decision making with probabilities

- **<u>Decision Tree</u>**



Example of decision analysis: ABC Restaurant (cont'd)
Decision making with probabilities

- <u>Expected Value For Each Decision</u>



EMV = .4(10,000) + .2(15,000) + .4(14,000)
= $12,600

EMV = .4(8,000) + .2(18,000) + .4(12,000)
= $11,600

EMV = .4(6,000) + .2(16,000) + .4(21,000)
= $14,000

Choose the model with largest EV, Model C.

Examples

- For the previous example of sports complex:
  - Suppose market research was conducted in the community where the complex will be built. This research allowed the company to estimate that the probability of low demand will be 0.35, and the probability of high demand will be 0.65. Which decision alternative should they select?

Example of decision analysis: sports complex
Decision making with probabilities

	STATES OF NATURE		
Alternatives	**Low**	**High**	
	(0.35)	**(0.65)**	**Expected value (EV)**
Small	8	8	$8(0.35) + 8(0.65) = 8$
Medium	5	15	$5(0.35) + 15(0.65) = 11.5$
Large	-11	22	$-11(0.35) + 22(0.65) = 10.45$

Recall that this is a profit payoff table. Thus since the decision to build a medium complex has the highest expected profit, this is our best decision.

Examples

- Expected Value of Perfect Information (EVPI)
  - Frequently information is available which can improve the probability estimates for the states of nature
  - EVPI is the increase in the expected profit that would result if one knew with certainty which state of nature would occur
  - EVPI provides an upper bound on the expected value of any sample or survey information

Examples

- EVPI calculation:
  - Step 1: Determine the optimal return corresponding to each state of nature
  - Step 2: Compute the expected value of these optimal returns
  - Step 3: Subtract the EV of the optimal decision from the amount determined in step (2)
- Example: ABC Restaurant
  - EVPI = .4(10,000) + .2(18,000) + .4(21,000) - 14,000 = $2,000

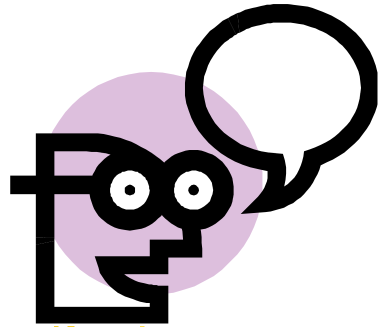
Further Reading

- Decision Theory - Decision Tables and Decision Trees, Game Theory
  - http://orms.pef.czu.cz/text/game-theory/DecisionTheory.html
- Decision trees -- Wikipedia
  - http://en.wikipedia.org/wiki/Decision_tree
- Mindtools: Decision Tree Analysis
  - http://www.mindtools.com/dectree.html

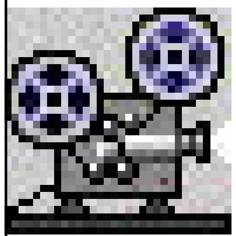
References

- Gilboa, I., 2011. *Making Better Decisions: Decision Theory in Practice*, Wiley-Blackwell, Chichester, West Sussex and Malden, MA. [658.403 G46]
- Peterson, M., 2009. *An Introduction to Decision Theory*, Cambridge University Press, New York. [519.542 P485 i61]

Interesting Talk



- TED Talk:



  - Dan Gilbert: Why we make bad decisions (33:35)
  - http://www.ted.com/talks/dan_gilbert_researches_happiness.html
  - Harvard psychologist Dan Gilbert presents research and data from his exploration of happiness -- sharing some surprising tests and experiments that you can also try on yourself