A study on the thermal comfort in sleeping environments in the subtropics—Developing a thermal comfort model for sleeping environments

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In the subtropics, air conditioning serves to maintain an appropriate indoor thermal environment not only in workplaces during daytime, but also at night for sleeping in bedrooms in residences or guestrooms in hotels. However, current practices in air conditioning, as well as the thermal comfort theories on which these practices are based, are primarily concerned with situations in which people are awake in workplaces at daytime. Therefore, these may not be directly applicable to air conditioning for sleeping environments. This paper reports on a theoretical study on a thermal comfort model in sleeping environments. A comfort equation applicable to sleeping thermal environments was derived by introducing appropriate modifications to Fanger’s comfort model. Comfort charts which were established by solving the comfort equation, and can be used for determining thermally neutral environmental conditions under a given bedding system have been developed. A related paper reports on an experimental study on measuring the total thermal insulation values of a wide range of bedding systems commonly used in the subtropics, which are an essential input to the comfort equation developed and reported in this paper.

Abstract

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Keywords: Thermal comfort; Sleeping thermal environment; Model; Comfort chart

1. Introduction

One of fundamental purposes for HVAC systems is to provide building occupants with comfortable thermal conditions. American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) defines thermal comfort as “the condition of the mind in which satisfaction is expressed with the thermal environment” [1]. The definition of thermal comfort leaves open as to what is meant by condition of mind or satisfaction, but it correctly emphasizes that the judgment of comfort is a cognitive process involving many inputs influenced by physical, physiological, psychological, and other factors. In general, comfort occurs when body temperatures are held within narrow ranges, skin moisture is low, and the physiological effort of regulation is minimized. Comfort also depends on behavioral actions such as altering clothing, altering activity, changing posture or location, changing the thermostat setting, opening a window, complaining, or leaving a space [2].

In the subtropics, air conditioning serves to maintain an appropriate indoor thermal environment not only in workplaces during daytime, but also at night for sleeping in bedrooms in residences or guestrooms in hotels. One fundamental issue in developing air conditioning technology is the theory on thermal comfort, whether for a workplace at daytime or a sleeping environment at night. Currently the theories on thermal comfort in workplaces at daytime are well established although there is still quite a lot need to research especially in the field of individual comfort and local body part discomfort/comfort and their
influence to overall comfort. However, research on thermal comfort for sleeping environments at night is limited and more work concerning the thermal comfort of people during sleep is needed, given that a human being spends approximately one-third of his/her life in sleep.

The last 50 years saw extensive research work on thermal comfort, covering many aspects related to thermal comfort. These include establishing models [3,4], and indices [5], carrying out experiments in climate chambers [3,6], and field surveys [7,8], establishing thermal comfort standards and evaluation methods [9,10], etc. In 1962, Macpherson defined the following six factors affecting thermal sensation: air temperature, air speed, humidity, mean radiant temperature, metabolic rate, and clothing levels [11,12]. Using the results from experiments with 1296 human subjects in a controlled climate chamber and a steady-state heat transfer model, Fanger developed an empirical comfort equation that incorporated the six factors [3]. Fanger related the comfort equation to the “Predicted Mean Vote” (PMV) index, which predicted the mean response of a large group of people according to the ASHRAE thermal sensation scale. The PMV was then incorporated into the “Predicted Percentage of Dissatisfied” (PPD) index, which predicted the percentage of the people who felt more than slightly warm or slightly cold (i.e., the percentage of the people who inclined to complain about the environment). Fanger’s PMV-PPD model on thermal comfort has been a path breaking contribution to the theory of thermal comfort and to the evaluation of indoor thermal environments in buildings. Both ISO Standard 7730 [13] and the new version of ASHRAE Standard 55 [1] include a short computer listing that facilitates the computing of PMV and PPD for a wide range of environmental and clothes resistance parameters.

In addition to Fanger’s PMV-PPD model, a two-node model, developed by Gagge et al. [5] at the J.B. Pierce Foundation Laboratory, Yale University (also known as the Pierce two-node model), was based on the heat balance equation developed by Stolwijk and Hardy [14], Gagge and Nishi [15]. Such a comfort model used a two compartment body structure dividing a body into two concentric cylinders—the inner cylinder for the body core whose
temperature, $t_{cr}$, is 37 °C, and the outer one for the skin layer whose temperature, $t_{sk}$, is 33 °C. The skin wettedness, a rationally derived physiological index defined as the ratio of the actual sweating rate to the maximum rate of sweating that would occur if the skin were completely wet, and skin temperature were incorporated into such a model to indicate the sensation of “comfort and discomfort” caused by perspiration [5].

However, almost all related studies were concerned with the situations where people were awake. The question that remains to be asked is then “can the existing thermal comfort standards be applied to sleeping environments in the subtropical regions?” Strictly speaking, the phase “thermal comfort” does not make too much sense for people during sleep. Based on the definition of thermal comfort, comfort is not a state condition, but rather a state of mind. The questions that need to be addressed include whether a human being can have any “mind” during sleep or not, and whether a human being can express his/her mind during sleep or not.

Nonetheless, there have been a few scattered research studies on the relationship between a sleeping thermal environment and the quality of sleep. Miyazawa demonstrated that the range of thermal neutral temperature was about 22±3 °C by using a questionnaire survey and measuring the air temperature and humidity in a bedroom as well as the esophageal temperature of five junior college students for a period of 214 days [16]. Miyazawa further showed that when the room air temperature ranged from 11 to 29 °C, the quality of sleep was not influenced remarkably. However, the bedroom was naturally ventilated rather than air conditioned. A study designed to examine the effect of sleeping thermal environment on sleep patterns by using electoencephalogram (EEG) with six men and six women sleeping at night in an experimental chamber where air temperatures were maintained at 10, 21.1, and 32.2 °C, respectively, was carried out [17]. The results also showed that the air temperature in a sleeping thermal environment did not affect the amount of time spent in various sleep stages. However, each subject slept only one night under the three temperatures respectively and only the air temperature was considered in the experimental study. It is interesting to note that both studies showed that under a rather wide range of air temperature (about 11–30 °C), air temperature would not have much effect on the quality of sleep. However, Haskell et al. reported that there were remarkable differences of individual’s sensitivity to air temperature during sleep. It was found that the sleep patterns of two subjects were similar when air temperature was maintained at 29 °C. However, the patterns differed significantly from each other when air temperature was maintained at 21 °C [18]. Although with only two subjects, this study indicated the observations that did not agree well with the results from the previous studies, implying that the individual’s sensitivity to sleeping thermal environment may have to be taken into account. Research work on the infant

sleeping thermal environment with its main focus on sudden infant death syndrome (SIDS) risk factors has been reported [19–21]. However, the sleeping thermal environment in these studies was a closed incubator, not a normal air-conditioned bedroom, and the research subjects were restricted to infants. In most of these previous studies, the number of human subject samples used was limited to single digits, in contrast to the 1296 subjects used by Fanger. This implies that using a statistically meaningful number of human subjects to systematically study thermal comfort in sleeping environments is hardly possible.

On sleep research, studies on the influence of air temperature to thermoregulation, metabolism, and the stages of sleep showed that during various sleep stages excluding rapid eye movement (REM) period, a human’s physiological response to room temperature changes was similar to that when a human being was awake [22,23]. Different researchers carrying out experimental studies on the effects of high and low ambient temperatures on human sleep stages adopted different thermoneutral temperature in sleep in the range of 20–32 °C [18,23–29]. Table 1 lists the details of the different thermoneutral temperature selected in various studies related to sleep. It can be seen that there is a fairly great range of the thermoneutral temperature, indicating that a well-agreed single thermoneutral temperature has not been established. In most of the research studies related to sleep, only ambient air temperature was referred to, but the mean radiant temperature or operative temperature and air velocity were not taken into account. In cases where 20–23 °C were selected, the test subjects were covered with bedding, while in others, the subjects may be in naked situation. This indicated that the relationship between the thermoneutral temperature and the insulation of bedding has not yet been established. Although the thermoneutral temperatures determined by researchers were different because of different experimental conditions (i.e. covered or naked), it can be clearly seen that the determined thermoneutral temperatures (20–22.2 °C at covered condition; 28–32 °C at naked condition) in sleeping thermal environments are different from the air temperature (24–26 °C) normally maintained in workplaces in summer.

The extensive previous work in establishing thermal comfort models in workplaces is expected to be useful to developing thermal comfort model for sleeping environments, as fundamentals such as the concept of thermal comfort and the heat transfer between a human body and its environment remain valid for non-workplace applications. On the other hand, Fanger’s PMV-PPD model has been widely used and accepted for design and field assessment of thermal comfort. Although the PMV-PPD model is useful only for predicting steady-state comfort responses while a two-node model can be used to predict physiological responses or responses to transient situations, a person sleeping in an air conditioned environment can be considered as being in a steady state and close to thermally
neutral. Therefore, Fanger’s PMV-PPD model has been adopted as a base model in this study, and is to be modified so that the application of such a model may be extended to sleeping thermal environments.

For instance, the clothing area factor \( f_{cl} \) included in the PMV-PPD model is meaningless when a body is lying on a bed [30]. Furthermore, it has been recommended in ISO Standard 7730 to use the PMV index when the metabolic rate \( M \) is between 46 and 232 W/m\(^2\) (0.8 and 4 met). The metabolic rate of a sleeping person is however 40 W/m\(^2\) (0.7 met), which is slightly out of the above range. On the other hand, it is pointed out in the new version of ASHRAE Standard 55 that when sleeping or resting with a reclining posture, a bed and bedding may provide considerable thermal insulation. It is hardly possible to determine the thermal insulation for most sleeping or resting situations unless a sleeping individual is immobile. Individuals will adjust bedding so as to suit their preferences. Provided that adequate bedding materials are available, the thermal environmental conditions desired for sleeping and resting vary considerably from person to person and cannot be determined by the PMV-PPD methods included in the ASHRAE Standard [1]. Therefore, it is necessary to modify Fanger’s PMV-PPD model for extending its application to sleeping environments.

This paper reports firstly on a theoretical study on thermal comfort model for sleeping environments. A comfort equation applicable to sleeping thermal environments was derived by introducing appropriate modifications to Fanger’s comfort equation. Secondly, it presents comfort charts which were established by solving the comfort equation developed, and can be used for determining thermally neutral environmental conditions under a given bedding system. Both PMV and PPD indexes can also be calculated for the purpose of assessing thermal comfort if and when necessary. Finally, the effect of the total insulation value provided by a bedding system on the comfort equation developed, for a wide range of bedding systems commonly used in the subtropics is reported.

2. Thermal comfort modeling for sleeping environments

Fanger [3], Hardy [32], Gagge and Nishi [15], and Gagge and Hardy [33] gave quantitative information on calculating the heat exchange between people and their environments. The mathematical descriptions of an energy balance equation and the statements for various terms of the heat exchange used in the heat balance equation are detailed in the ASHRAE Handbook of Fundamentals [2].

2.1. Energy balance of a human body

Fig. 1 shows the thermal interaction between a human body and its environment. The total metabolic rate of work \( M \) produced within the body is the metabolic rate required for the person’s activity plus the metabolic rate required for shivering. A portion of the body’s energy production may be expended as the external work done by muscles \( W \). The net heat production in the human body \( M - W \) is either stored \( S \), causing the body’s temperature to rise, or dissipated to the environment through skin surface \( q_{sk} \) and respiratory tract \( q_{res} \). Therefore, the heat balance for a human body is

\[
M - W = q_{sk} + q_{res} + S = (C + R + E_{sk}) + (C_{res} + E_{res}) + (S_{sk} + S_{cr}). \tag{1}
\]

All the terms in Eq. (1) have the units of power per unit area and refer to the surface area of a nude body. The most useful measure of nude body surface area, originally proposed by DuBois and DuBois [2], is described by

\[
A_D = 0.202m^{0.425}l^{0.725}. \tag{2}
\]

A human body can be considered as consisting of two concentric thermal compartments, the skin and the core [14]. The rate of heat storage in the body can be written separately for each compartment in terms of thermal capacity and change rate of temperature in each

<table>
<thead>
<tr>
<th>Literature</th>
<th>Thermoneutral temperature ( (^\circ C) )</th>
<th>Condition of test subjects</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karacan et al., 1978 [25]</td>
<td>22.2</td>
<td>Covered</td>
<td>Air temperature</td>
</tr>
<tr>
<td>Haskell et al., 1981 [18]</td>
<td>29</td>
<td>Naked</td>
<td>Air temperature</td>
</tr>
<tr>
<td>Candas et al., 1982 [23]</td>
<td>32</td>
<td>Naked</td>
<td>Operative temperature</td>
</tr>
<tr>
<td>Palca, 1986 [27]</td>
<td>29</td>
<td>Naked</td>
<td>Air temperature</td>
</tr>
<tr>
<td>Sewitch et al., 1986 [26]</td>
<td>20–22</td>
<td>Covered</td>
<td>Air temperature</td>
</tr>
<tr>
<td>Di Nisi et al., 1989 [28]</td>
<td>30</td>
<td>Naked</td>
<td>Operative temperature</td>
</tr>
<tr>
<td>Dewasmes et al., 2000 [29]</td>
<td>28</td>
<td>Naked</td>
<td>Air temperature</td>
</tr>
</tbody>
</table>

Table 1
Different thermoneutral temperatures adopted in studies related to sleep.
compartment, as follows:

\[ S_{ct} = \frac{(1 - \alpha_{sk})mc_{ph}d_{ct}}{A_{D}} \frac{dT}{dT}, \]  

(3)

\[ S_{sk} = \frac{\alpha_{sk}mc_{ph}d_{sk}}{A_{D}} \frac{dT}{dT}. \]  

(4)

2.2. Thermal exchanges between a human body and its environment

2.2.1. Sensible heat loss from skin

Sensible heat exchange from skin surface to a surrounding environment must pass through clothing. Both convective and radiative heat losses from the outer surface of a clothed body can be expressed in terms of a heat transfer coefficient and the difference between the mean temperature of the outer surface of the clothed body and an appropriate environmental temperature:

\[ C = f_{cl}h_{c}(t_{cl} - t_{a}), \]  

(5)

\[ R = f_{cl}h_{r}(t_{cl} - \bar{t}_{r}). \]  

(6)

The coefficients, \( h_{c}, \) and, \( h_{r}, \) are both evaluated at the clothing surface. Eqs. (5) and (6) are commonly combined to describe the total sensible heat exchange by these two heat exchange mechanisms in terms of an operative temperature, \( t_{o}, \) and a combined heat transfer coefficient, \( h: \)

\[ C + R = f_{cl}h(t_{cl} - t_{o}), \]  

(7)

where

\[ t_{o} = \frac{h_{t}t_{t} + h_{c}t_{a}}{h_{t} + h_{c}}, \]  

(8)

\[ h = h_{t} + h_{c}. \]  

(9)

Based on Eq. (8), the operative temperature, \( t_{o}, \) can be defined as the average of the mean radiant and ambient air temperatures, weighted by their respective heat transfer coefficients. The actual transport of sensible heat passing through clothing involves conduction, convection, and radiation. It is usually convenient to combine these into a single thermal resistance of clothing, \( R_{cl} \)

\[ C + R = (\bar{t}_{sk} - t_{o})/R_{cl}. \]  

(10)

Combining Eqs. (7) and (10) to eliminate \( t_{cl}: \)

\[ C + R = \frac{\bar{t}_{sk} - t_{o}}{R_{cl} + 1/(f_{cl}h)}. \]  

(11)

2.2.2. Evaporative heat loss from skin

Evaporative heat loss from skin, \( E_{sk}, \) depends on the amount of moisture on skin and the difference between the water vapor pressure at skin surface and that in the ambient environment:

\[ E_{sk} = w(p_{sk,s} - p_{a})/R_{ecl} + 1/(f_{cl}h). \]  

(12)

Evaporative heat loss from skin is a combination of the evaporation of sweat secreted due to thermoregulatory control mechanisms, \( E_{res}, \) and the natural diffusion of water through skin, \( E_{dif}: \)

\[ E_{sk} = E_{res} + E_{dif}. \]  

(13)

Theoretically, the maximum possible evaporative heat loss from a skin surface, \( E_{max}, \) occurs when the skin surface is completely wet (i.e., the skin wettedness, \( w, \) is equal to 1). The skin wettedness is the ratio of the actual evaporative heat loss to the maximum possible evaporative heat loss, \( E_{max}: \)

\[ w = E_{sk}/E_{max}. \]  

(14)
Skin wettedness is important in determining evaporative heat loss. It ranges from about 0.06 caused by $E_{\text{diff}}$ alone (i.e., with no regulatory sweating) for normal conditions, to 1.0 when theoretically a skin surface is totally wet with perspiration, a condition that occurs rarely in practice. For large values of $E_{\text{max}}$ or long exposures to low humidity, the value of $w$ may drop to as low as 0.02, since dehydration of outer skin layers alters their diffusive characteristics.

### 2.2.3. Respiratory losses

Respiratory heat loss, $q_{\text{res}}$, is often expressed in terms of sensible heat loss, $C_{\text{res}}$, and latent heat loss, $E_{\text{res}}$. Sensible loss ($C_{\text{res}}$) and latent loss ($E_{\text{res}}$) due to respiration are relatively small compared to the other terms in Eq. (1) and can be estimated, respectively, by the following equations [2]:

\[
C_{\text{res}} = 0.0014M(34 - t_a),
\]

\[
E_{\text{res}} = 0.0173M(5.87 - p_a).
\]

### 2.3. Assumptions and modifications adopted for sleeping environments

Eqs. (1)–(16) are normally applicable to sedentary or near sedentary physical activity levels, e.g. typical office work. Assumptions and modifications are needed if these equations are to become applicable to sleeping environments.

For a sleeping person in a reclining posture with a specific bedding system which consists of a bed and mattress, bedding and sleepwear, it is assumed that the sleeping person is immobile during the whole period of sleep, therefore,

\[
M = 40 \text{ W/m}^2,
\]

\[
W = 0 \text{ W/m}^2.
\]

For a bedding system rather than clothing, the intrinsic clothing resistance, $R_{\text{cl}}$, in Eq. (11), cannot be determined because the clothing area factor, $f_{\text{cl}}$, is meaningless when a body is lying on a bed. Therefore, Eq. (11) can be rearranged in terms of the total thermal resistance ($R_t$) provided by a bed, pillow, bedding, sleepwear and the air layer surrounding a human body so that the intrinsic clothing resistance, $R_{\text{cl}}$, and the clothing area factor, $f_{\text{cl}}$, may be substituted by $R_t$, as follows:

\[
R_t = R_{\text{cl}} + 1/(hf_{\text{cl}}) = R_{\text{cl}} + R_s/f_{\text{cl}},
\]

\[
C + R = \frac{\bar{t}_{\text{sk}} - t_o}{R_t}.
\]

Similarly, Eq. (12) can be rewritten to become

\[
E_{\text{sk}} = \frac{w(p_{\text{sk},s} - p_a)}{R_{\text{cl}}},
\]

According to the Lewis relation

\[
i_mL_R = \frac{R_t}{R_{\text{cl}}},
\]

Combining Eqs. (21) and (22) to eliminate $R_{\text{cl}}$:

\[
E_{\text{sk}} = \frac{i_mL_R^w(p_{\text{sk},s} - p_a)}{R_t},
\]

where Lewis ratio ($L_R$) equals approximately to 16.5 K/kPa at typical indoor conditions [2].

Since the purpose of the thermoregulatory system in a human body is to maintain an essentially constant internal body temperature, it can be assumed that for long exposure (for periods not less than 15 minutes as specified in ASHRAE Standard 55) to a constant sleeping thermal environment with a constant ($M$–$W$), a heat balance will exist for the human body (steady state condition). In other words, there will be no significant heat storage within the body. Therefore, Eqs. (3) and (4) can be changed to

\[
S_{\text{ct}} = \frac{(1 - z_{\text{sk}})mc_{p,b} \frac{dt_{\text{ct}}}{dt}}{A_D} = 0,
\]

\[
S_{\text{sk}} = \frac{z_{\text{sk}}mc_{p,b} \frac{dt_{\text{sk}}}{dt}}{A_D} = 0.
\]

Based on all the assumptions and modifications introduced above for sleeping environments, Eq. (1) can be rewritten to become:

\[
40 = \frac{\bar{t}_{\text{sk}} - t_o}{R_t} + \frac{i_mL_R^w(p_{\text{sk},s} - p_a)}{R_t} + 0.056(34 - t_a)
\]

\[+ 0.692(5.87 - p_a).
\]

### 2.4. Conditions for thermal comfort in sleeping environments

A basic condition for thermal comfort in sleeping environments is that thermal neutrality is achieved during sleep. Thermal neutrality for a person is defined as the condition in which the subject would prefer neither warmer nor cooler surroundings. Obviously the first requirement for thermal comfort in sleeping environments is that the heat balance equation (26) be satisfied. However, heat balance alone is not sufficient to achieve thermal comfort. In a wide range of environmental conditions where heat balance can be obtained, thermal comfort may be achieved only within a narrow range of the conditions. The following linear regression equations indicate values of $t_{\text{sk}}$ and $E_{\text{res,ct}}$ that provide thermal comfort, which were proposed as the second and third conditions for optimal thermal comfort by Fanger [3]:

\[
t_{\text{sk,req}} = 35.7 - 0.0275(M - W),
\]

\[
E_{\text{res,ct,req}} = 0.42(M - W - 58.15).
\]

It can be seen from the two equations that in a state of physiological thermal neutrality during sedentary ($M = 58.15 \text{ W/m}^2$, $W = 0$), the mean skin temperature is
around 34 °C and there is no regulation of body temperature by sweating (i.e., sweating does not occur). The skin temperature necessary for comfort falls, and moderate sweating takes place at a higher activity level. However, in a state of thermal neutrality for a sleeping person whose activity level is lower than sedentary (\( M = 40 \text{ W/m}^2 \), \( W = 0 \)), the mean skin temperature would increase and sweating would either not occur \( (E_{\text{sw,req}} \text{ is meaningless when less than zero}) \). Therefore, the second and third conditions for thermal comfort in a sleeping environment may be changed to

\[
t_{sk,\text{req}} = 35.7 - 0.0275(M - W) = 34.6(^\circ \text{C}),
\]

\[E_{\text{sw,req}} = 0.\]

With no regulatory sweating for normal conditions, the skin wettedness \( (w) \) equals to 0.06, caused by \( E_{\text{sw}} \) alone [5]:

\[w = 0.06.\]

### 2.5. Comfort equation for sleeping environments

Combining the three conditions (i.e., Eqs. (26), (29) and (31)) for thermal comfort in a sleeping environment to obtain

\[40 = \frac{34.6 - t_o}{R_t} + \frac{0.06 im L_R (p_{sk,s} - p_a)}{R_t} + 0.056 (34 - t_o) + 0.692 (5.87 - p_a).
\]

In order to solve Eq. (32), some of its parameters such as the heat transfer coefficients, \( h_c, h_r, \) and, \( h_t, \) and the permeation efficiency, \( i_m, \) need to be determined. ASHRAE Handbook of Fundamentals [2] provides the necessary data and methods used to calculate these parameters.

The radiant heat transfer coefficient, \( h_r, \) is nearly constant for typical indoor temperatures, and a value of 4.7 W/(m² K) is sufficient for most calculations [2]. The convective heat transfer coefficient, \( h_c, \) for reeling persons can be calculated by [34]

\[h_c = 2.7 + 8.7 v^{0.67} \text{ for } 0.15<v<1.5,
\]

\[h_c = 5.1 \text{ for } 0<v\leq0.15.
\]

Quantitative values of \( h_c \) are important, not only in estimating convection loss, but in evaluating operative temperature, \( t_o.\)

An experimental study by McCullough et al. has suggested that the permeation efficiency, \( i_m, \) of ensembles worn indoors generally fell in the range of 0.3–0.5, and that assuming \( i_m = 0.38 \) is reasonably accurate for most applications although \( i_m \) for a given clothing ensemble is a function of the environment as well as the clothing properties [35]. Since the properties of bedding are likely to be similar to that of clothing, such a value \( (i_m = 0.38) \) can also be adopted for a bedding system.

According to the properties of saturated water/steam, the water vapor partial pressure in saturated air, \( p_{sk,sv}, \) when \( t_{sk} = 34.6 ^\circ \text{C} \), is

\[p_{sk,sv} = 5.52 \text{(kPa)}.
\]

Using Eqs. (8), (35) and \( i_m = 0.38, \) \( L_R = 16.5 \text{K/kPa}, \) \( h_t = 4.7 \text{W/(m}^2\text{K)}, \) a comfort equation for sleeping environments, which combines both environmental and personal variables to produce a thermal neutral sensation, may be derived from Eq. (32) as follows:

\[40 = \frac{1}{R_t} \left[ \left(34.6 - \frac{4.7 h_t + h_t t_o}{4.7 + h_t} \right) + 0.3762 (5.52 - p_a) \right]
\]

\[+ 0.056 (34 - t_o) + 0.692 (5.87 - p_a).
\]

Obviously the satisfaction of the comfort equation (36) means that the three comfort conditions are met at the same time since it combines the three equations for thermal comfort in sleeping environments. There are five variables, \( R_t, t_o, t_a, p_a, \) and \( h_t \) in Eq. (36). The convective heat transfer coefficient, \( h_c, \) is the function of air velocity, \( v, \) therefore, four variables (i.e., \( t_o, t_a, p_a, \) and \( v \)) are thermal environmental variables. The total thermal resistance, \( R_t, \) is the function of a number of variables such as bedding, sleepwear, bed and mattress, the percentage coverage of body surface area by bedding and bed, air velocity, direction of airflow, and posture, etc. How these factors would influence the total thermal resistance of a bedding system will be discussed in a related paper [31].

### 2.6. PMV and PPD for sleeping environments

Fanger suggested a PMV index which predicted the mean response of a large group of people according to the ASHRAE thermal sensation scale [3]. He related PMV to the imbalance between the actual heat flow from a human body in a given environment and the heat flow required for optimum comfort at a specified activity by the following equation:

\[\text{PMV} = [0.303 \exp(-0.036M) + 0.028]L = zL,
\]

where \( L \) is the thermal load on the body, defined as the difference between internal heat production and heat loss to the environment for a person hypothetically kept at comfort values of \( t_{sk} \) and \( E_{\text{sw}} \) at the activity level. Thermal load, \( L, \) is therefore the difference between the left- and right-hand sides of Comfort Equation calculated for the given values of the environmental conditions.

After estimating the PMV using Eq. (37), the PPD with a given condition can also be estimated by

\[\text{PPD} = 100 - 95 \exp[-(0.03353 \text{PMV}^4 + 0.2179 \text{PMV}^2)],
\]

where dissatisfaction is defined as anybody not voting \(-1, +1, \) or 0. A PPD of 10% corresponds to the PMV range of \( \pm 0.5, \) and even with PMV = 0, about 5% of the people are dissatisfied.

Fanger’s PMV-PPD model is widely used and accepted for design and field assessment of thermal comfort.
However, the sensitivity coefficient, $a$, used in the PMV equation (37) requires to be experimentally evaluated over a wide range of conditions. The $a$ value in Eq. (37) was obtained based on the results from experiments with 1296 human subjects with different metabolic rate ($\geq 58.15 \text{ W/m}^2$) in controlled climate chambers. For sleeping environments, it is hardly possible to carry out experiments with a statistically meaningful number of human subjects for quantifying the sensitivity coefficient. Furthermore, it is not possible either to conduct a questionnaire survey during the course of sleep. A survey may only be conducted after sleep when however a tested human subject sample may not be able to accurately relate his/her sleep quality to his/her thermal environment. Therefore, it has to be assumed that the sensitivity coefficient, $a$, obtained by Fanger is also applicable to sleeping environments although the relatively low activity level (sleep) was not included in the experiments. In other words, extrapolation was applied to extend the range of metabolic rate down to 40 W/m$^2$ (0.7 met when sleeping). Hence, the PMV for a sleeping environment can be calculated by

$$\text{PMV} = 0.0998 \left\{ 40 - \frac{1}{R_t} \left[ \left( 34.6 - \frac{4.7t_e + h_c t_u}{4.7 + h_c} \right) \right] \right\}$$

$$+ 0.3762(5.52 - p_a)$$

$$- 0.0998[0.06(34 - t_a) + 0.692(5.87 - p_a)]. \quad (39)$$

The PPD for a sleeping environment can then be determined by Eq. (38).

In Eq. (36), $R_t$ is the total thermal resistance for a bedding system. It will have to be determined by experimental approach. A related paper reports on an experimental work on measuring the total resistance of the bedding systems commonly used in the subtropics in southern China including Hong Kong [31].

### 3. Solving comfort equation and establishing comfort charts

There are a large number of combinations of variables (i.e., environmental variables such as air temperature, air velocity, and the total resistance value of a bedding system) that may satisfy the thermal Comfort Equation for sleeping environments. A FORTRAN program has been developed and run in the platform of Microsoft Fortran PowerStation to solve the Comfort Equation (36) for various combinations of variables. Comfort charts (Figs. 2–5) have been established, and can be used for determining thermally

Fig. 2. Comfort lines (operative temperature vs. wet bulb temperature) with an air velocity not greater than 0.15 m/s.

Fig. 3. Relationship between operative temperature and the total insulation value with an air velocity not greater than 0.15 m/s.
neutral environmental conditions under a given bedding system. Furthermore, if and when necessary, both PMV and PPD indexes can also be calculated for the purpose of assessing thermal comfort.

Fig. 2 illustrates the comfort lines showing the combinations of operative temperature, wet bulb temperature and the total insulation value, under which thermal neutrality can be achieved. It can be seen from the figure that the influence of relative humidity on the thermal comfort of a sleeping person is relatively small. A change from absolutely dry air (RH = 0%) to saturated air (RH = 100%) can be compensated by only a 1.63–1.95 °C (at the range of 2.4–0.8 clo total insulation values) decrease of operative temperature. The higher the total insulation value, the less the decrease of operative temperature. For example, when the total insulation value is 1.0 clo, a 1.93 °C reduction of operative temperature will adequately compensate the change of relative humidity from 0% to 100%, compared to a 1.7 °C reduction of operative temperature at the total insulation of 2.0 clo.

It should be noted that the comfort lines in Fig. 2 include the entire humidity range from 0% to 100%. Although there is no reason, from a thermal comfort point of view, to avoid extreme relative humidity, there exist other reasons for avoiding the extremes. For example, it is recommended in ASHRAE Standard 62 [36] that the relative humidity in habitable spaces preferably be maintained between 30% and 60% to minimize the growth of allergenic or pathogenic organisms. Similar to the ASHRAE Standard, ISO Standard 7730 also recommends that the relative humidity be kept between 30% and 70%. The limits are set to decrease the risk of unpleasantly wet or dry skin, eye irritation, static electricity, microbial growth and respiratory diseases.

On the other hand, the total insulation value provided by a bedding system would significantly influence the thermal neutral temperature for sleeping persons. This can be more clearly illustrated by Fig. 3, which shows the relationships between the thermal neutral temperature and the total thermal insulation of a bedding system under different indoor relative humidity levels. It can be seen in Fig. 3, for example, at 50% relative humidity when the total insulation value increases from 1.0 to 2.0 clo, the thermal neutral temperature will decrease from 29.5 to 24.2 °C. It can also be seen from Fig. 3 that a linear relationship between operative temperature and the total insulation value is demonstrated. Fig. 3 also provides answers to the question which is related to the thermoneutral temperature during sleep and raised in Introduction. In other words, the relationship between the thermoneutral temperature and the total insulation value provided by a bedding system has been established. For example, the thermoneutral temperature would be 29.6 °C at 50% relative humidity for a naked sleeping person with the total insulation value of a bedding system being 0.98 clo. This agrees well with the range of 28–32 °C for thermoneutral temperature at naked conditions in the various earlier studies related to sleep.

Figs. 4 and 5 illustrate the comfort lines showing the relationships between air temperature and mean radiant temperature at different air velocities, at the total insulation value of 1.0 and 2.0 clo, respectively, both at 50% relative humidity. The comfort lines illustrate at different air velocities the different combinations of air temperature, $t_a$, and mean radiant temperature, $t_r$, which satisfy the Comfort Equation (36). For example, if $I_T = 1.0$ clo, RH = 50% (Fig. 4) and $v < 0.15$ m/s, an environmental
condition with $t_a = 28.0^\circ C$ and $t_r = 32.7^\circ C$ may achieve thermal comfort. However, with the same $I_T = 1.0$ clo and RH $= 50\%$, when $v = 1.0$ m/s, an environmental condition with $t_a = 29.1^\circ C$ and $t_r = 32.7^\circ C$ (or $t_a = 28.0^\circ C$ and $t_r = 35.7^\circ C$) may also achieve thermal comfort.

It can be seen in both figures that the comfort lines crossed each other at the same point where air temperature equals to the mean radiant temperature and that a linear relationship between air temperature and mean radiant temperature is demonstrated. On the other hand, although the comfort lines in Figs. 4 and 5 include a wide range of air velocity from 0 to 1.5 m/s, in practice the air velocity is limited to not greater than 0.15 m/s for air-conditioned sleeping environments. There are two reasons for this. The first is that higher velocity may cause draft (a local convective cooling) and local thermal discomfort since people are more thermally sensitive and consequently the risk of local discomfort is higher during sleep with lower metabolic rates and/or with less insulation. The second is that even for daytime applications, the precise relationship between increased air speed and improved comfort is yet to be established although an elevated air velocity may be used to increase the maximum temperature for acceptable comfort if an affected occupant is able to control the air velocity. However, an occupant is obviously not able to control the air velocity when sleeping.

4. Discussions

4.1. The effect of the total insulation of a bedding system on thermal comfort

By solving the Comfort Equation (36), it can be seen that the total thermal insulation value of a bedding system significantly affects the thermal neutral temperature for sleeping environments and is therefore an important variable in the Comfort Equation. The slope of the comfort line at 50\% relative humidity in Fig. 3 is $-0.189$ clo/\(^\circ C\), indicating that a decrease of only $0.189$ clo in $I_T$ would result in an increase of $1^\circ C$ of optimum operative temperature in order to maintain the thermal neutrality for sleeping. Therefore, during sleeping when the metabolic rate is $0.7$ met, the effect of changing $I_T$ on the optimum operative temperature is approximately $5.3^\circ C$ per clo. This is similar to the effect of changing clothing’s insulation on the optimum operative temperature for sedentary activity ($6.0^\circ C$ per clo).

4.2. Air conditioning culture

Individuals may adjust their bedding so as to suit individual preferences when sleeping. A person’s dressing/bedding code may further be affected by the local custom and culture in addition to personal preference. However, the culture and habits of dressing codes in Hong Kong (a typical subtropical city) have led to the use of a relatively low indoor air temperature for daytime air conditioning. The Cantonese translation of the term of air-conditioning is literally wrong meaning “Cold Air” when air-conditioning was first introduced to Hong Kong. Since then, the notion that an air-conditioned space has to be “cold” has been deeply implanted into the minds of occupants. Even when energy conservation became a design issue due to the energy crisis in 1973, the call for a lower indoor air temperature still prevailed. Dressing/bedding (i.e., suit and mattress) codes with a relatively high clo value (high insulation) in summer has consequently become a culture in offices and residences. This may partly help explain the results of a previous study [37], which showed that most of the local people would prefer a relatively low indoor air temperature at below 24\(^\circ C\), and about 20\% of the respondents would prefer even a lower indoor air temperature at below 20\(^\circ C\) during sleep. On the other hand, they would wear suits rather than shirts/shorts in offices, and use mattresses, wear sleepwear and cover themselves with quilts or blankets during sleep.

The sensation of a human being is the “trigger” for him/her to change his/her own preference to suit the environment, or to change the environment to suit him/her. If a relatively low indoor air temperature is pre-maintained in a sleeping environment, an occupant will therefore use a bedding system of a higher insulation value to suit the environment. However, this kind of air-conditioning culture contradicts to the principle of energy conservation. From the energy conservation view of point, people should use as less sleepwear and bedding (or cover as less body area by bedding) as possible to lower the total thermal insulation of a bedding system. Consequently, a higher indoor air temperature in bedrooms at night may be maintained for thermal comfort. Therefore, it is necessary to encourage people to change their habits of using sleepwear and bedding (i.e., change the local air-conditioning culture) in order to save energy and their costs for maintaining a suitable sleeping thermal environment could also be reduced considerably.

5. Conclusions

One fundamental issue in developing air conditioning technology is the theory on thermal comfort. However, research on thermal comfort for sleeping environments at night is limited. A theoretical study on thermal comfort for sleeping environments has been conducted and reported in this paper. The results of the study may be potentially used as the basis for establishing criteria on thermal comfort for sleeping environments.

A comfort equation applicable to sleeping thermal environments has been derived by introducing appropriate assumptions and modifications to Fanger’s comfort model. The comfort equation for sleeping environments contains five variables: air temperature, mean radiant temperature, water vapor pressure in ambient air, air velocity and the total resistance/insulation provided by a bedding system.
Based on the derived comfort equation for sleeping environments, the relationship between the thermonutral temperature and the total insulation value provided by a bedding system has been established. The comfort charts with different comfort lines have also been developed, and can be used for determining thermally neutral environmental conditions under a given bedding system. Furthermore, both PMV and PPD indexes can also be calculated for the purpose of assessing thermal comfort, if and when necessary.

The effect of changing the total thermal insulation of a bedding system on the optimum operative temperature is approximately 5.3 °C per clo, indicating that the total insulation value provided by a bedding system significantly affects the thermal neutral temperature for sleeping persons.

Locally, the air conditioning culture of having dressing/bedding with a relatively high clo value (higher insulation) in summer, and at the same time, maintaining a relatively low indoor air temperature, should be reviewed and changed. People should use as less sleepwear and bedding (or cover as less body surface area by bedding) as possible to lower the total insulation of a bedding system. This would result in a relatively higher indoor air temperature maintained in bedrooms without losing thermal comfort at night, and consequently, reduced energy use for air conditioning for sleeping environments.

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References

[14] Stolwijk JAJ, Hardy JD. Control of body temperature. In: Handbook of physiology, reaction to environment agents, American Physiological Society, 1977 [Chapter 4].
[16] Miyazawa M. Seasonal changes of sleep environment at bedtime and on arising. The proceeding of the 18th symposium on human-environment system 1994 [In Japanese].
[27] Telliez F. Skin temperature and sleep architecture in response to a mild cold stress in women. The proceeding of the 18th symposium on human–environment system 1994 [In Japanese].

