

# BBSE3009 Project Management and Engineering Economics

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## Decision making among alternatives



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# Contents

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- Evaluate Engineering Projects
- Present-Worth Analysis
- Comparing Mutually Exclusive Projects
- Annual Equivalence Analysis
- Rate of Return Analysis
- Benefit-Cost Analysis

# Evaluate Engineering Projects

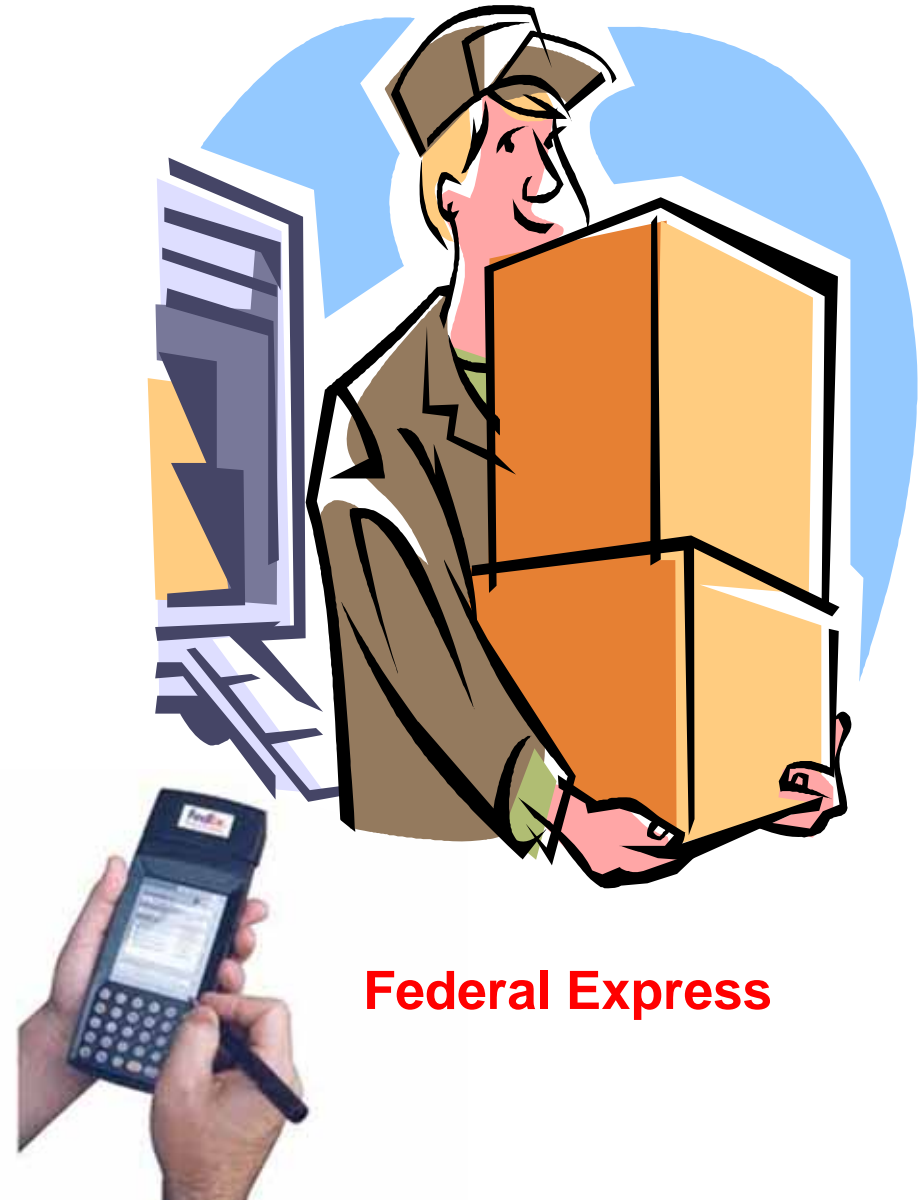
- Evaluate business and engineering assets payback period
  - Loan versus Project Cash Flows
  - Initial Project Screening Methods
  - Present-Worth Analysis
  - Methods to Compare Mutually Exclusive Alternatives



# Example – Federal Express

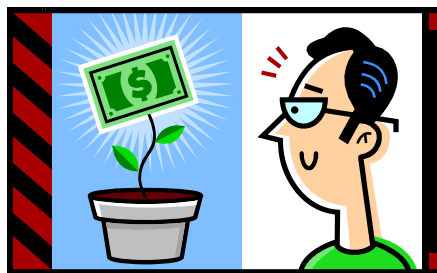
## Nature of Project:

- Equip 40,000 couriers with Power Pads
- Save 10 seconds per pickup stop
- Investment cost: \$150 million
- Expected savings: \$20 million per year



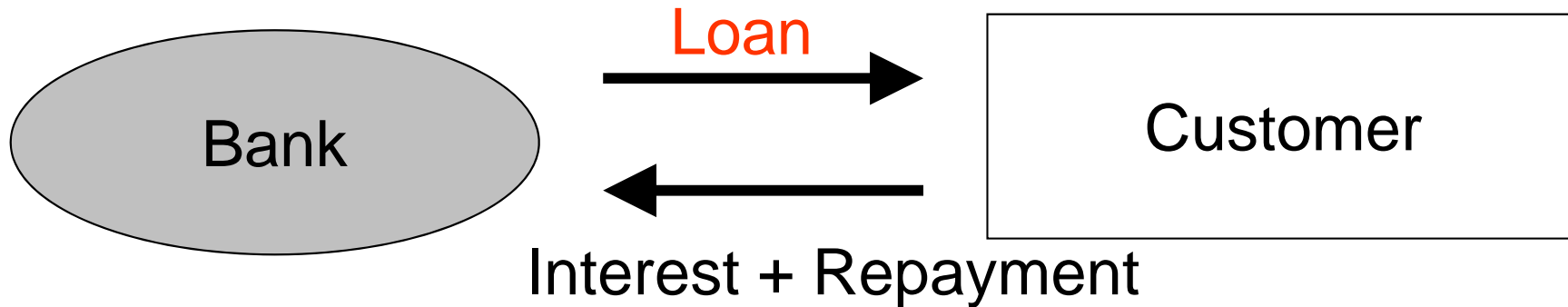
# Ultimate Questions

- Is it worth investing \$150 million to save \$20 million per year, say over 10 years?
- How long does it take to recover the initial investment?
- What kind of interest rate should be used in evaluating business investment opportunities?

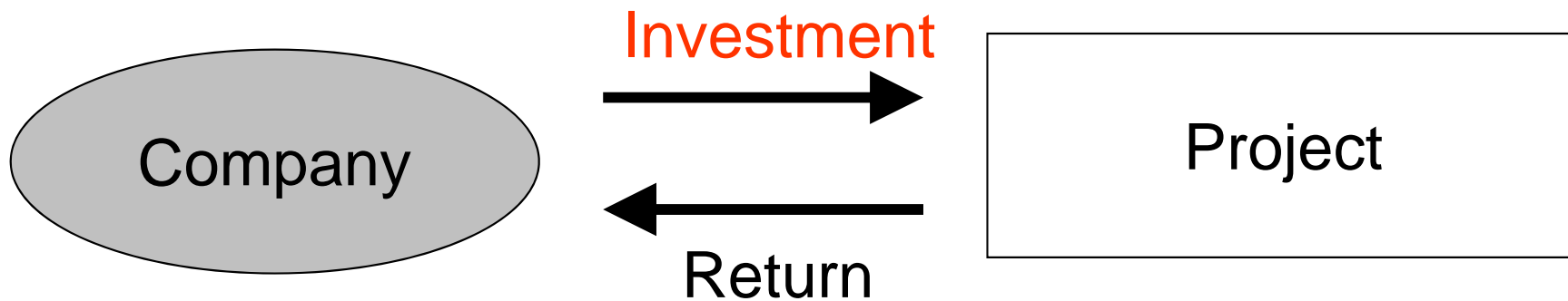


# Bank Loan vs. Investment Project

## Bank Loan (loan cash flow)



## Investment Project (project cash flow)



# Initial Project Screening Method

**Payback Method** screens projects on the basis of how long it takes for net receipts to equal investment spending.

**Principle:**

How fast can I recover my initial investment?

**Method:**

Based on cumulative cash flow (or accounting profit)

**Screening Guideline:**

If the payback period is less than or equal to some specified payback period, the project would be considered for further analysis.

**Weakness:**

Does not consider the time value of money

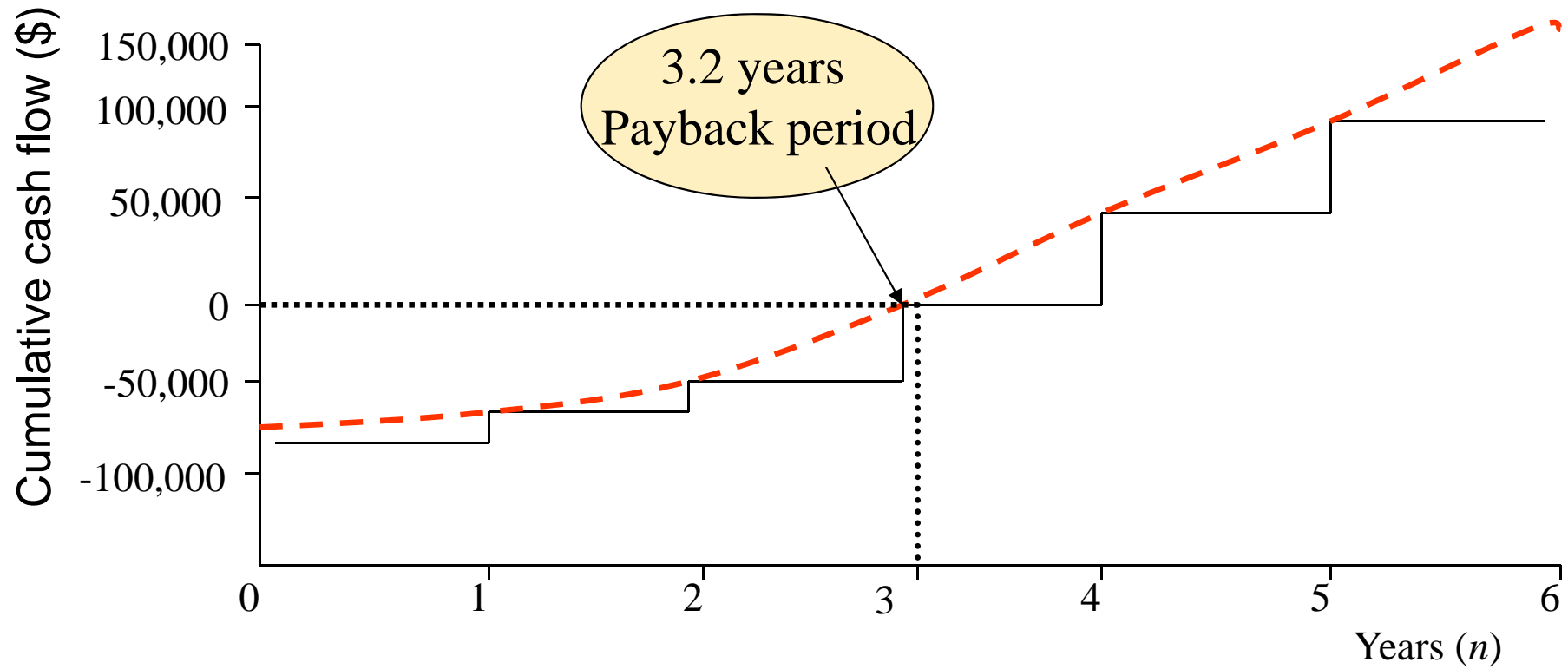
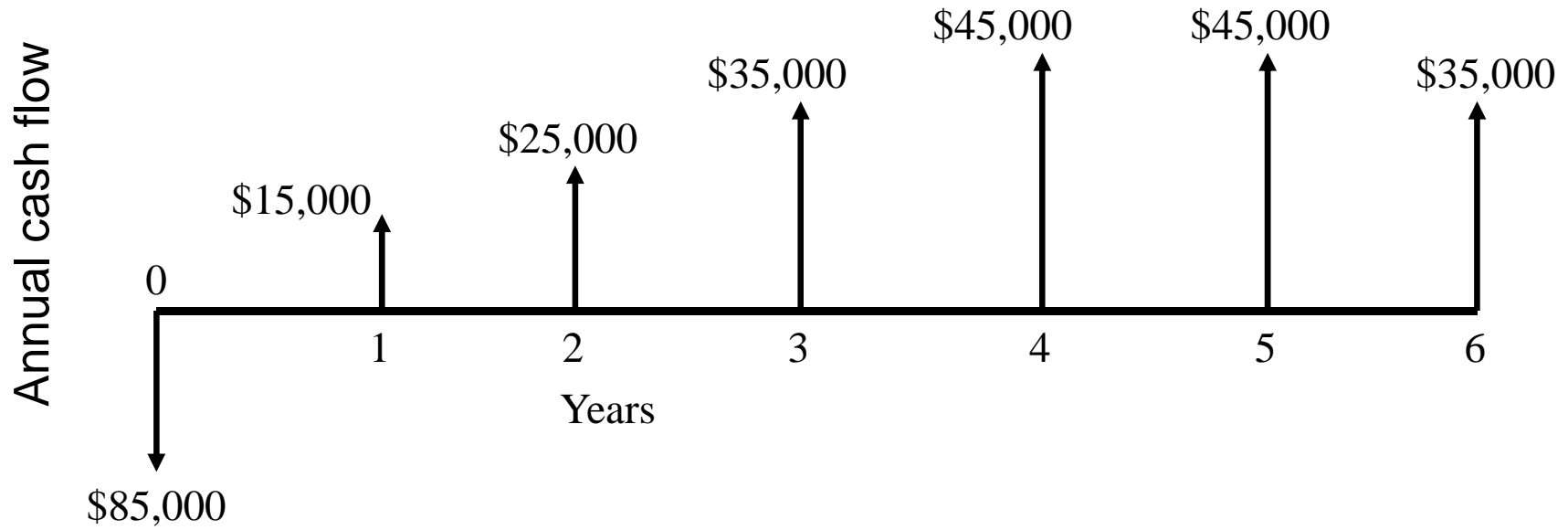
# Example: Conventional Payback Period with Salvage Value

<u><i>N</i></u>	<u>Cash Flow</u>	<u>Cum. Flow</u>
0	-\$105,000+\$20,000	-\$85,000
1	\$35,000	-\$50,000
2	\$45,000	-\$5,000
3	\$50,000	\$45,000
4	\$50,000	\$95,000
5	\$45,000	\$140,000
6	\$35,000	\$175,000



Payback period should occur somewhere between  $N = 2$  and  $N = 3$ .





# Discounted Payback Period

## ❑ Principle:

How fast can I recover my initial investment plus interest?

## ❑ Method:

Based on the cumulative discounted cash flow

## ❑ Screening Guideline:

If the discounted payback period (DPP) is less than or equal to some specified bench-mark period, the project could be considered for further analysis.

## ❑ Weakness:

Cash flows occurring after DPP are ignored

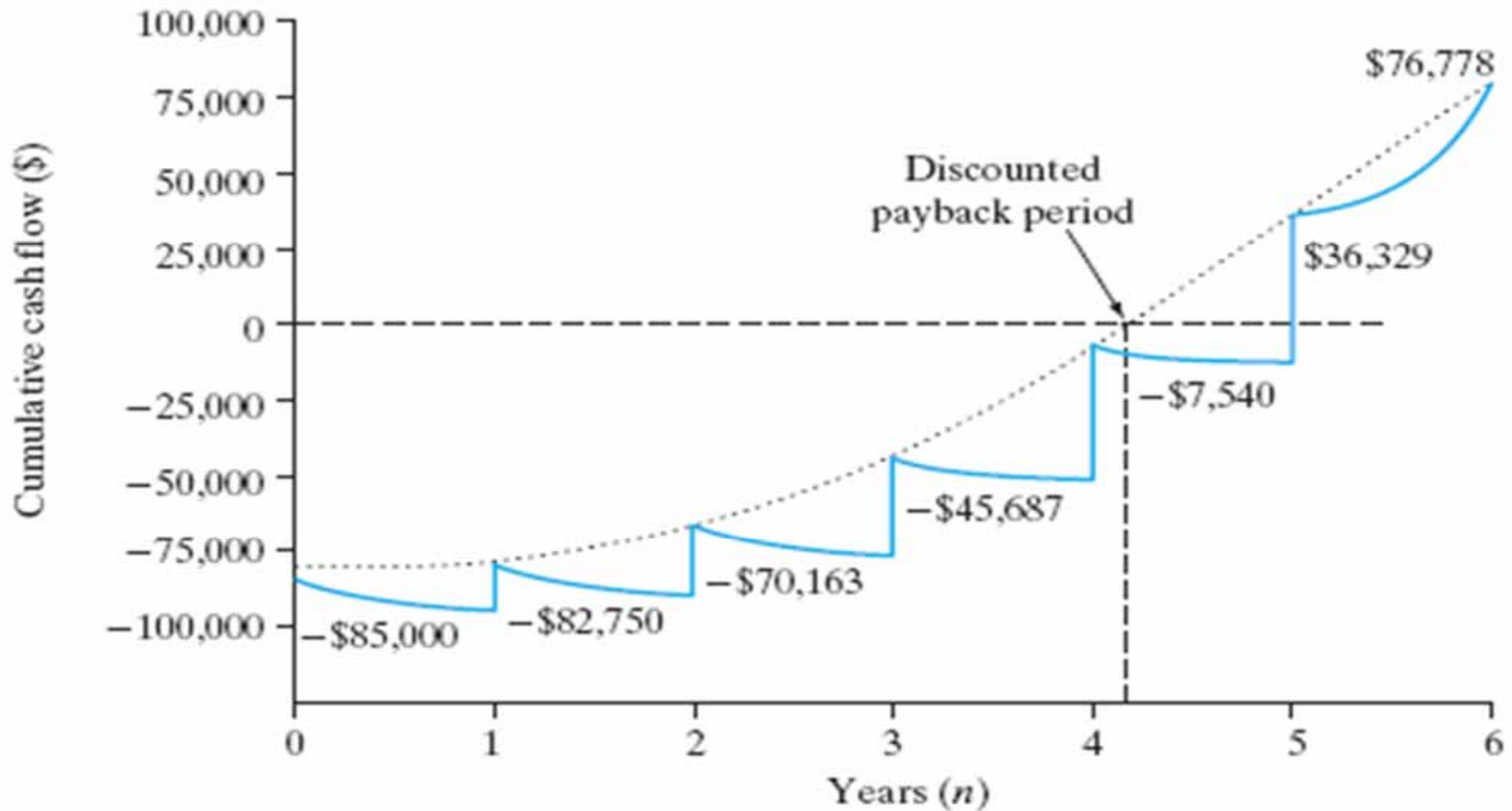
# Example: Discounted Payback Period Calculation

Modify the procedure and consider time value of money, such as the cost of money (interest) used to support the project.

Period ( $n$ )	Cash Flow ( $A_n$ )	Cost of Funds (15%)*	Ending Cash Balance
0	-\$85,000	0	-\$85,000
1	15,000	$-\$85,000(0.15) = -\$12,750$	-82,750
2	25,000	$-\$82,750(0.15) = -12,413$	-70,163
3	35,000	$-\$70,163(0.15) = -10,524$	-45,687
4	45,000	$-\$45,687(0.15) = -6,853$	-7,540
5	45,000	$-\$7,540(0.15) = -1,131$	36,329
6	35,000	$\$36,329(0.15) = 5,449$	76,778

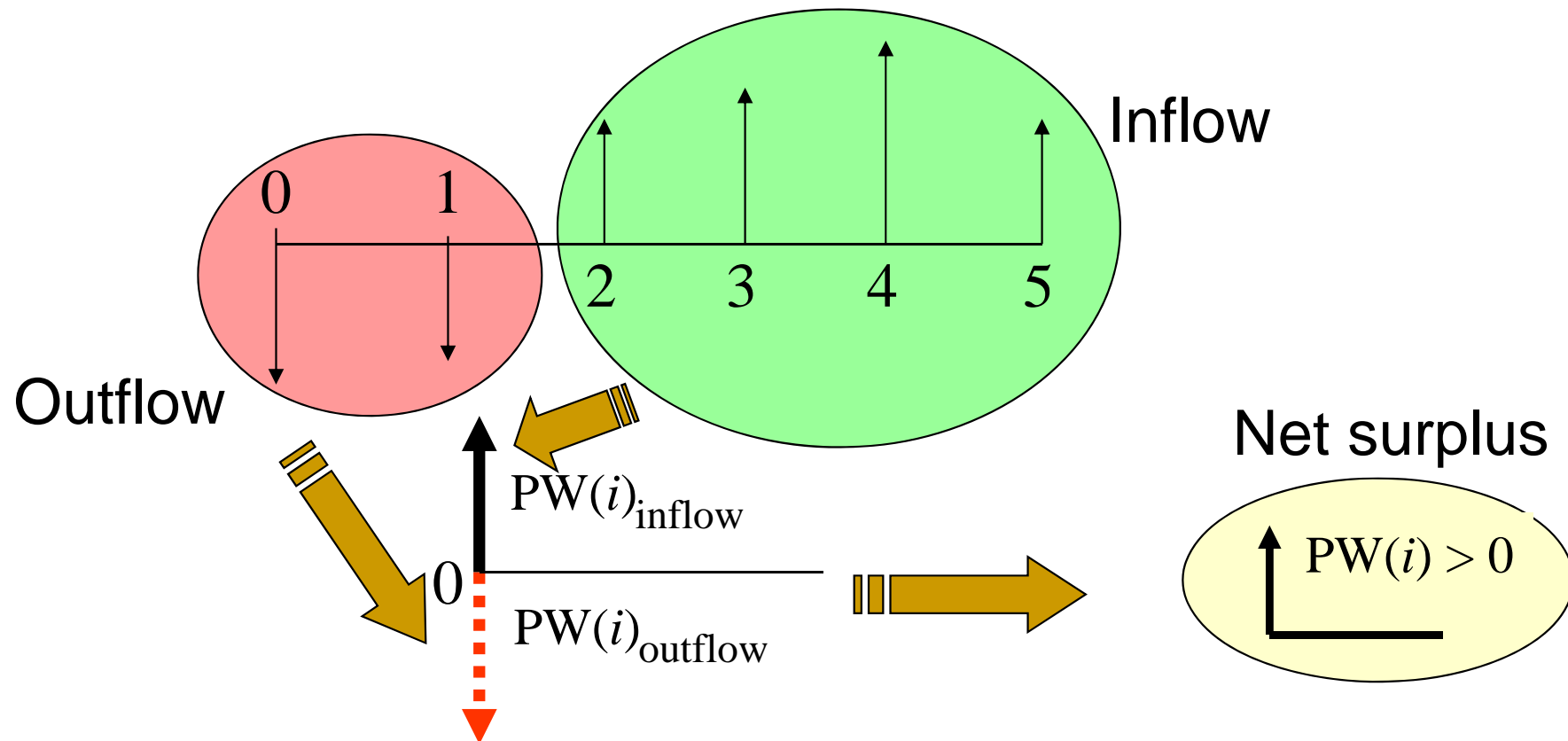
\* Cost of funds = (Unrecovered beginning balance) X (interest rate)

# Illustration of Discounted Payback Period



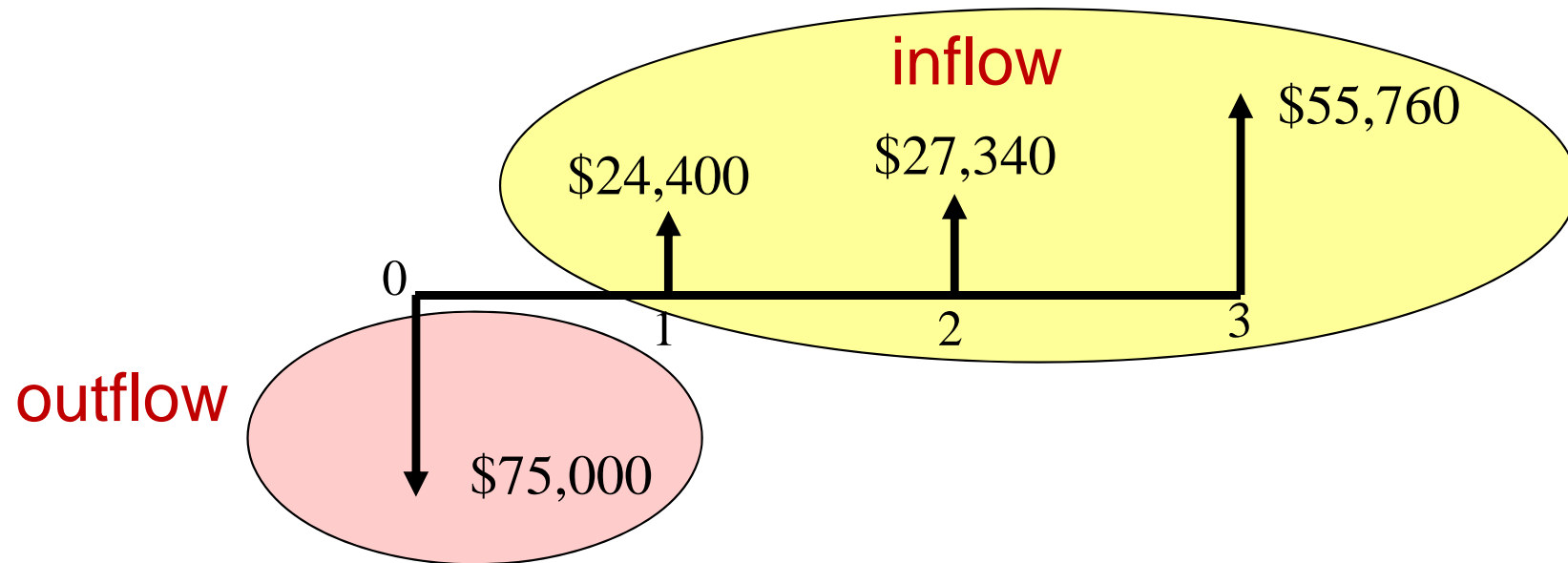
# Net Present Worth (NPW) Measure

- ❑ **Principle:** Compute the equivalent net surplus at  $n = 0$  for a given interest rate of  $i$ .
- ❑ **Decision Rule for Single Project Evaluation:** Accept the project if the net surplus is positive.
- ❑ **Decision Rule for Comparing Multiple Alternatives:** Select the alternative with the largest net present worth.



# Example: Tiger Machine Tool Company

## Net Present Worth – Uneven Flows



$$\begin{aligned}
 PW(15\%)_{\text{inflow}} &= \$24,400(P/F, 15\%, 1) + \$27,340(P/F, 15\%, 2) \\
 &\quad + \$55,760(P/F, 15\%, 3) \\
 &= \boxed{\$78,553}
 \end{aligned}$$

$$PW(15\%)_{\text{outflow}} = \boxed{\$75,000}$$

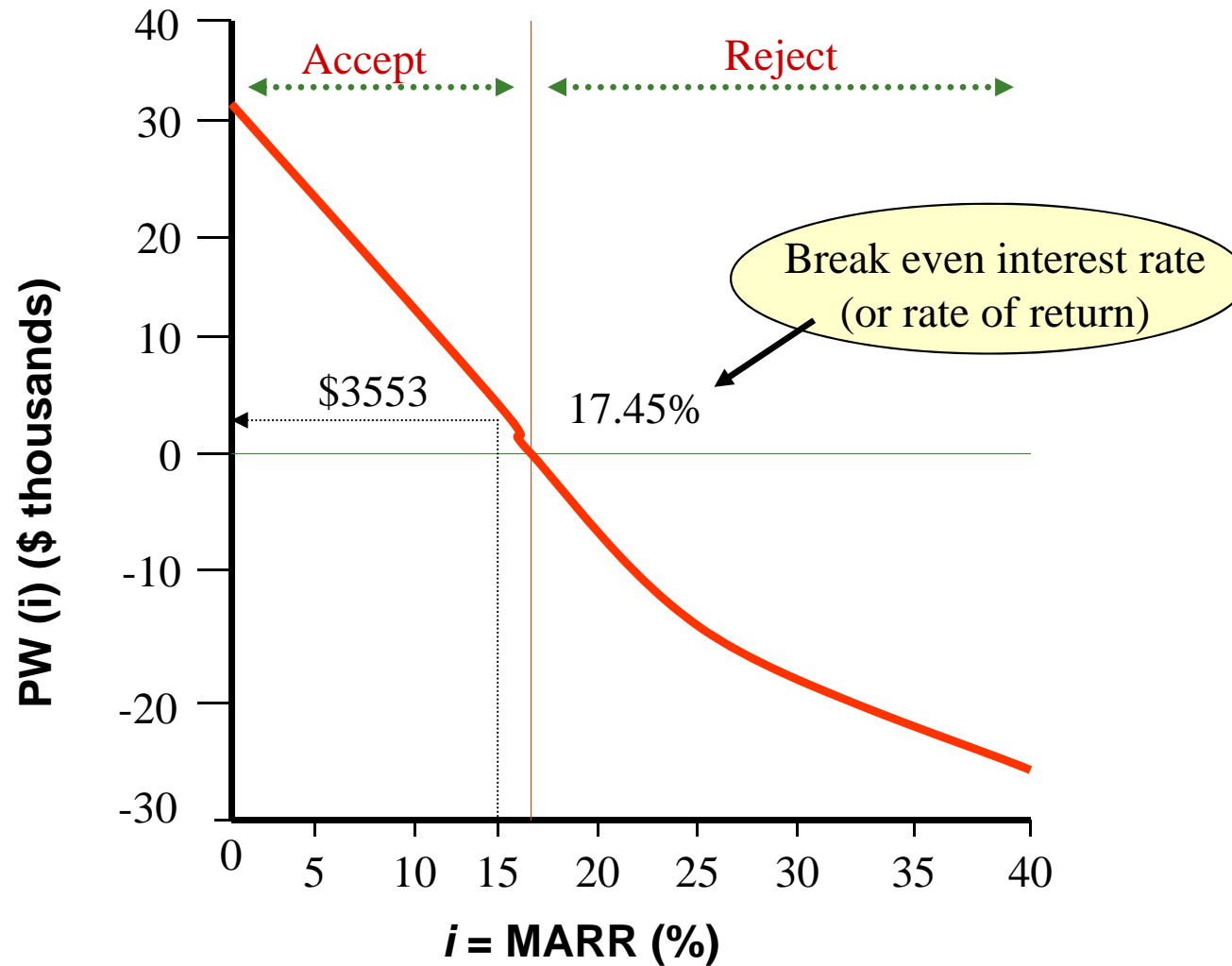
$$\begin{aligned}
 PW(15\%) &= \$78,553 - \$75,000 \\
 &= \boxed{\$3,553 > 0, \text{ Accept}}
 \end{aligned}$$

## Present Worth Amounts at Varying Interest Rates

$i$ (%)	PW( $i$ )	$i$ (%)	PW( $i$ )
0	\$32,500	20	-\$3,412
2	27,743	22	-5,924
4	23,309	24	-8,296
6	19,169	26	-10,539
8	15,296	28	-12,662
10	11,670	30	-14,673
12	8,270	32	-16,580
14	5,077	34	-18,360
16	2,076	36	-20,110
17.45*	0	38	-21,745
18	-751	40	-23,302

\*Break even interest rate

# Present Worth Profile





# Present Worth Analysis

- What does Present Worth (PW) really mean?
  - 1. Project Balance Concept
  - 2. Investment Pool Concept



# Project Balance Concept

- Suppose that the firm has no internal funds to finance the project, so will borrow the entire investment from a bank at an interest rate of 12%
- Then, any proceeds from the project will be used to pay off the bank loan
- Then, our interest is to see if how much money would be left over at the end of the project period

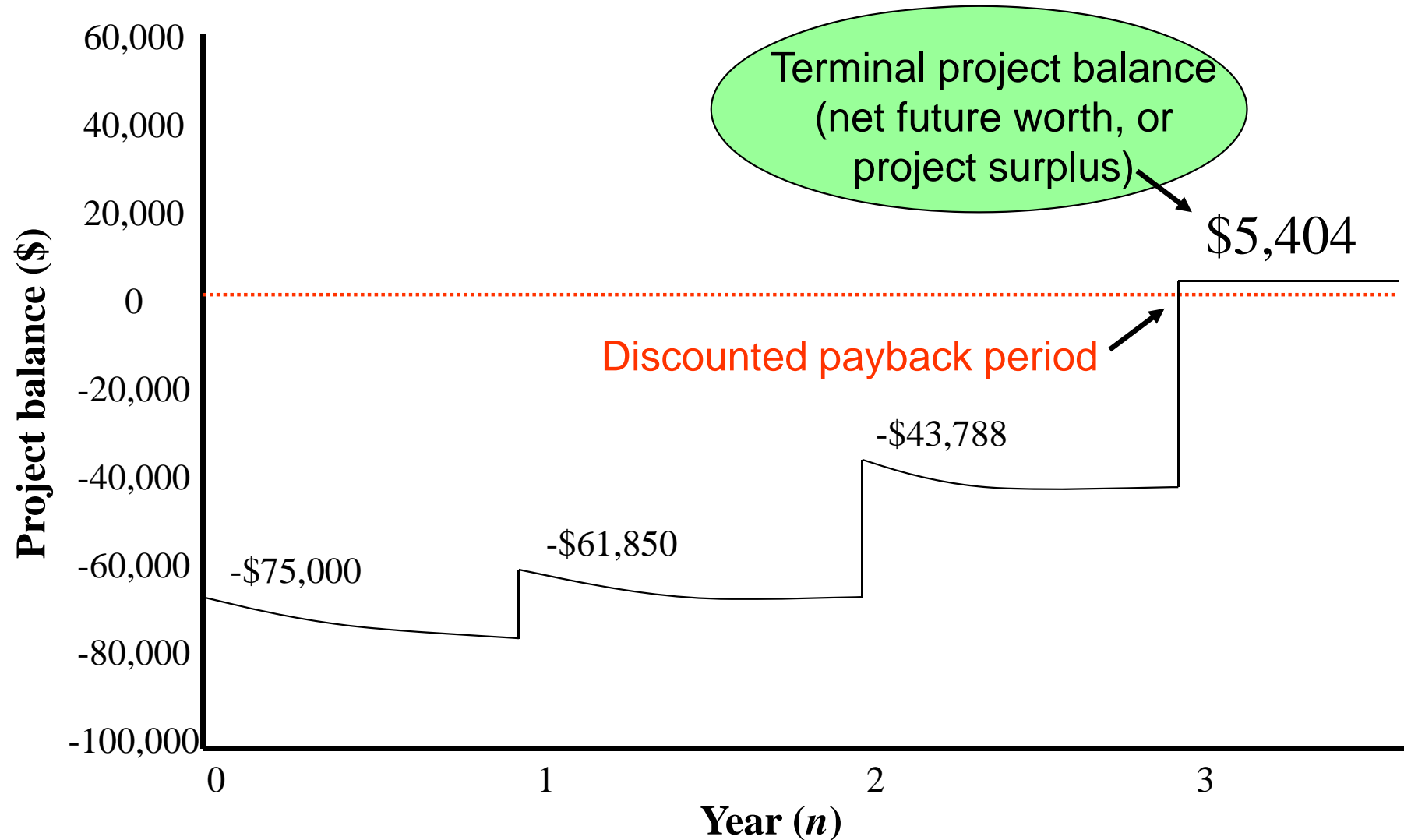
# Project Balance Concept (Bank loan)

<i>N</i>	0	1	2	3
<b>Beginning Balance</b>		-\$75,000	-\$61,850	-\$43,788
<b>Interest</b>		-\$11,250	-\$9,278	-\$6,568
<b>Payment</b>	-\$75,000	+\$24,400	+\$27,340	+\$55,760
<b>Project Balance</b>	-\$75,000	-\$61,850	-\$43,788	+\$5,404

Net future worth, FW(15%)

**PW(15%) = \$5,404 (P/F, 15%, 3) = \$3,553**

# Project Balance Diagram



## Four pieces of information:

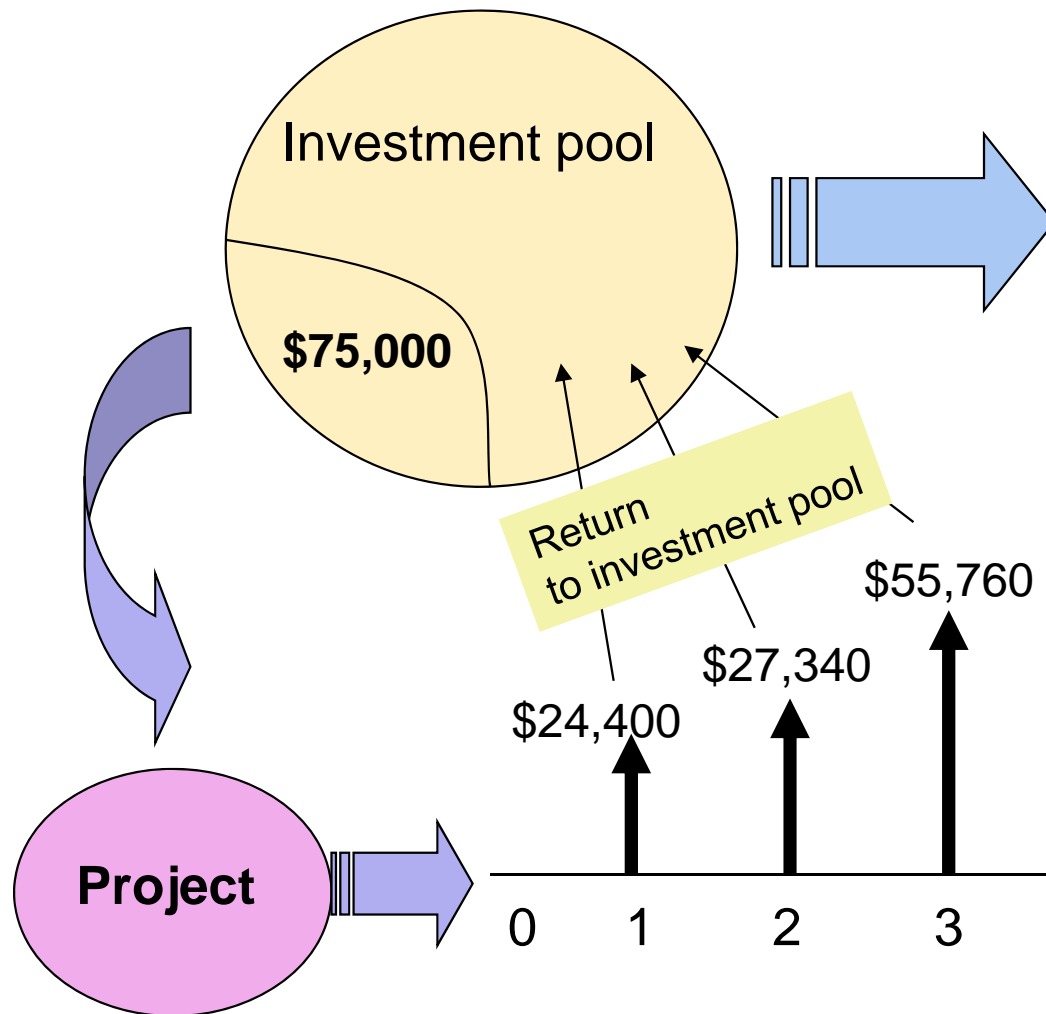
1. The exposure to financial risk;
2. The discounted payback period;
3. The profit potential;
4. The net future worth

# Investment Pool Concept

- Suppose the company has \$75,000. It has two options.
  - (1) Take the money out and invest it in the project or
  - (2) leave the money in the pool and continue to earn a 12% interest.
    - If Option 1 is taken, any proceeds from the project will be returned to the investment pool and earn 12% interest yearly until the end of the project period
- Let's see what the consequences are for each option.

# Meaning of Net Present Worth

$N = 3$



How much would you have if the Investment is made?

$$\$24,400(F/P, 15\%, 2) = \$32,269$$

$$\$27,340(F/P, 15\%, 1) = \$31,441$$

$$\$55,760(F/P, 15\%, 0) = \underline{\$55,760}$$

**NFW of the project \$119,470**

How much would you have if the investment was not made?

$$\$75,000(F/P, 15\%, 3) = \$114,066$$

What is the net gain from the investment?

$$\$119,470 - \$114,066 = \$5,404$$

$$\mathbf{PW(15\%) = \$5,404(P/F, 15\%, 3) = \$3,553}$$

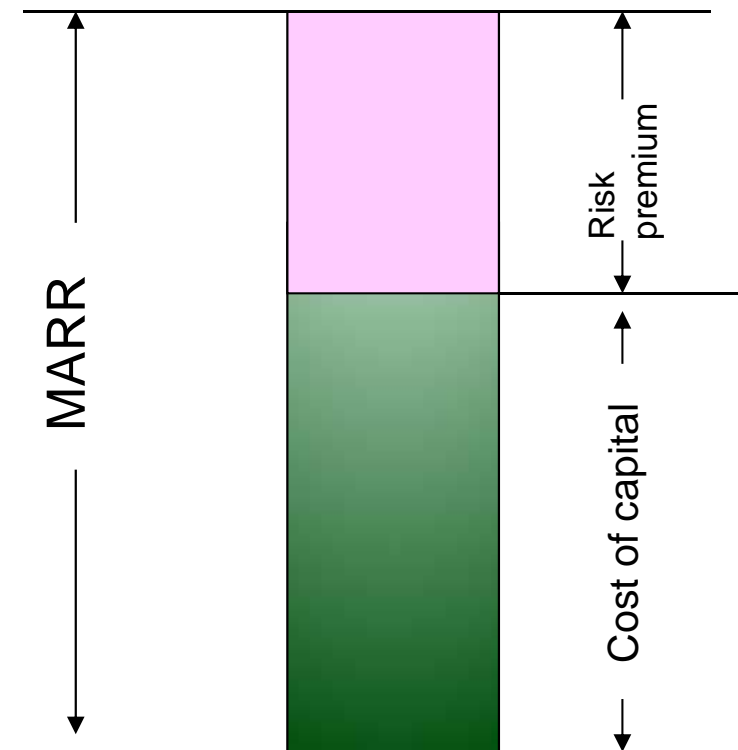
# What Factors Should the Company Consider in Selecting a MARR in Project Evaluation?

## ■ Cost of capital

- The required return necessary to make an investment project worthwhile.
- Viewed as the rate of return that a firm would receive if it invested its money someplace else with a similar risk

## ■ Risk premium

- The additional risk associated with the project if you are dealing with a project with higher risk than normal project



(MARR = minimum attractive rate of return)

# Practice Problem

- An electrical motor rated at 15HP needs to be purchased for \$1,000.
- The service life of the motor is known to be 10 years with negligible salvage value.
- Its full load efficiency is 85%.
- The cost of energy is \$0.08 per kWh.
- The intended use of the motor is 4,000 hours per year.
- Find the total present worth cost of owning and operating the motor at 10% interest.



# Solution

□  $1\text{HP} = 0.7457\text{kW}$

□  $15\text{HP} = 15 \times 0.7457 = 11.1855\text{kW}$

□ Required input power at 85% efficiency rating:

$$\frac{11.1855\text{kW}}{0.85} = 13.1594\text{kW}$$

□ Required total kWh per year

$$13.1594\text{kW} \times 4,000 \text{ hours/year} = 52,638 \text{ kWh/yr}$$

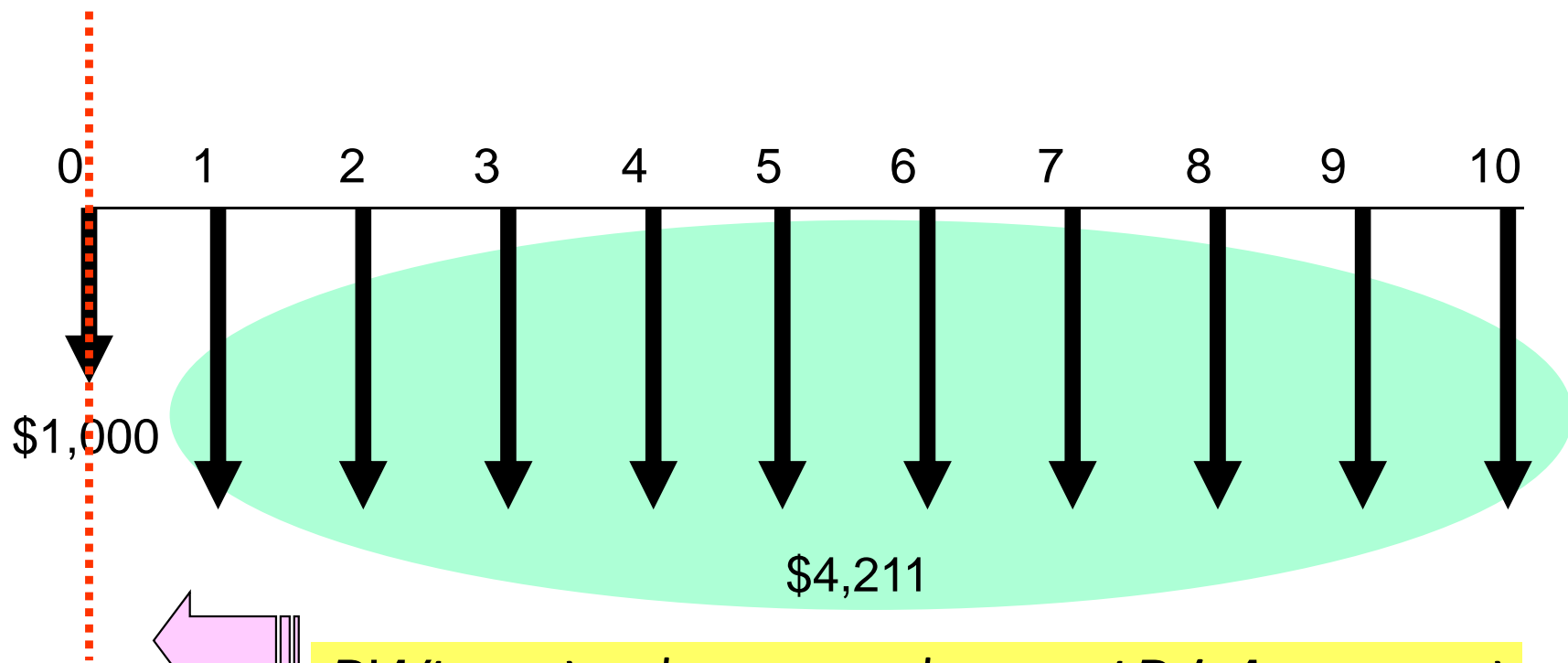
□ Total annual energy cost to operate the motor

$$52,638\text{kWh} \times \$0.08/\text{kWh} = \$4,211/\text{yr}$$

□ The total present worth cost of owning and operating the motor

$$\begin{aligned} PW(10\%) &= \$1,000 + \$4,211(P/A, 10\%, 10) \\ &= \$26,875 \end{aligned}$$

# Cash Flow Series Associated with Owning and Operating the Motor



$$PW(10\%) = \$1,000 + \$4,211(P/A, 10\%, 10)$$
$$= \$26,875$$

# Comparing Mutually Exclusive Projects

## ❑ Mutually Exclusive Projects

➤ When alternatives are mutually exclusive, any one of the alternatives will fulfill the same need, and the selection of one alternative implies that the others will be excluded. Example of buying versus leasing car.

## ❑ Alternative vs. Project

➤ When we use terms **alternative** and **project** interchangeably to mean decision option.

## ❑ Do-Nothing Alternative

➤ When considering an investment, we are in one of two situations: Either the project is aimed at replacing an existing asset or system, or it is a new attempt.

➤ If a process or system already in place to accomplish our business objectives is adequate, then we must determine which, if any, new proposals are economical replacement. If none are feasible, then we do nothing.

➤ If the existing system has failed, then the choice among proposed alternatives is mandatory (i.e., do nothing is not an option).

# Comparing Mutually Exclusive Projects

## Revenue Projects

are projects that generate revenues that depend on the choice of alternative that we want to select the alternative with the largest net gains

## Service Projects

are projects that generate revenues that do not depend on the choice of project, but must produce the same amount of output (revenue) with lower production cost

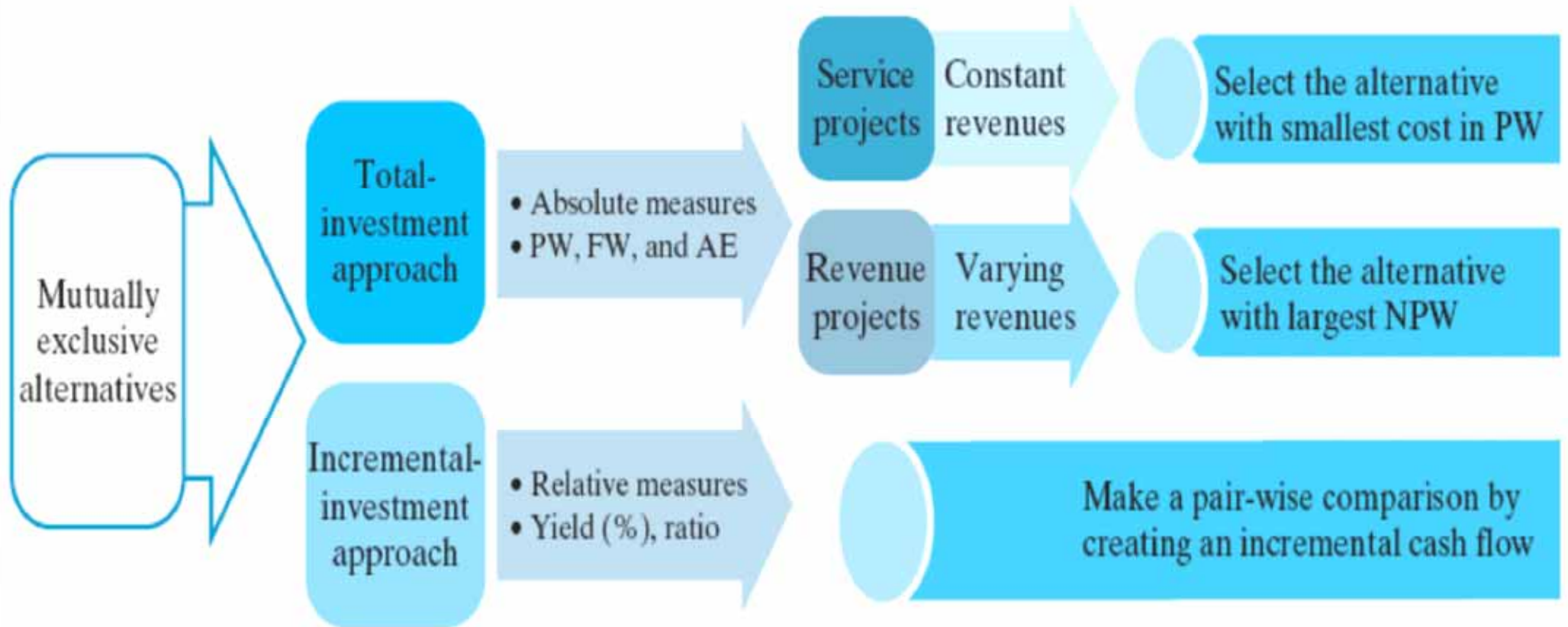
# Comparing Mutually Exclusive Projects

## Analysis Period

is the time span over which the economic effects of an investment will be evaluated.

- The study (analysis) period, sometimes called the *planning horizon*, is the selected time period over which Mutually Exclusive alternatives are compared.
- Factors influence the decision are; the required service period, the useful life of the shorter lived alternative, the useful lived of the longer lived alternative, company policy and so on.
- Consider Analysis period to be **Required Service Period**.
- One convenient choice of analysis is the period of the useful life of the investment project.

# Road Map – A Process of Making a Choice among Mutually Exclusive Alternatives



# Comparing Mutually Exclusive Projects

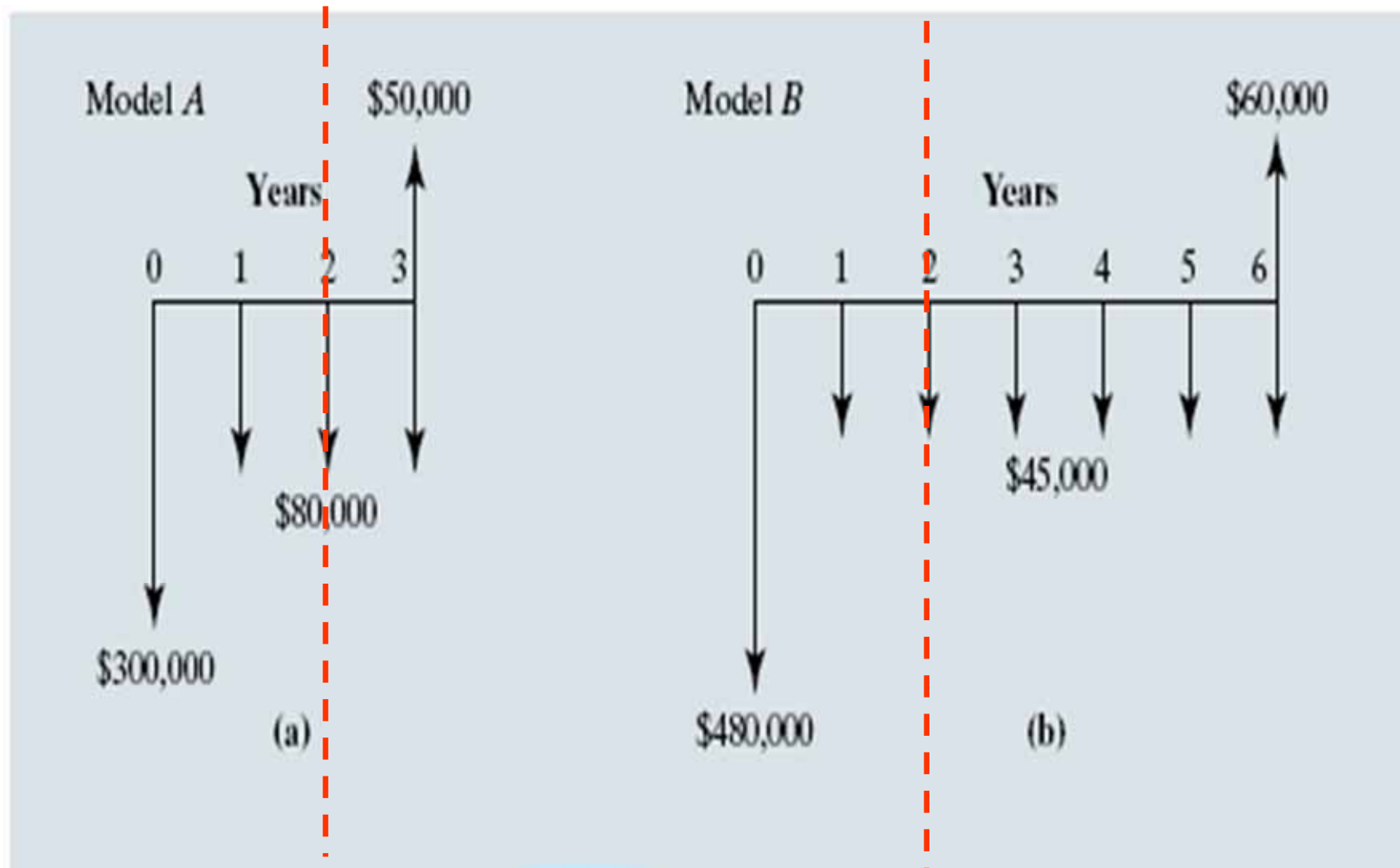
- ❑ **Principle:** Projects must be compared over an **equal time** span.
  
- ❑ **Rule of Thumb:** If the required service period is given, the analysis period should be the same as the required service period.
  
- **Case 1:** Project lives longer than the analysis period
- **Case 2:** Project lives shorter than the analysis period

# Case 1: Project lives longer than the analysis period

- Estimate the **salvage value** at the end of required service period
- Compute the PW for each project over the required service period

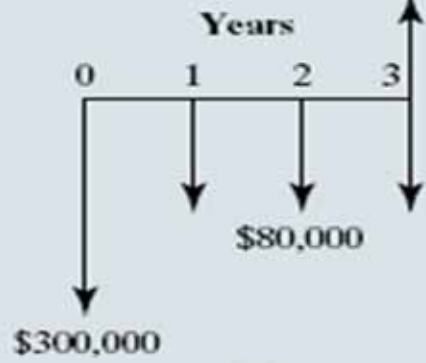


# Example: Comparison of unequal-lived service projects when the required service period is shorter than the individual project life



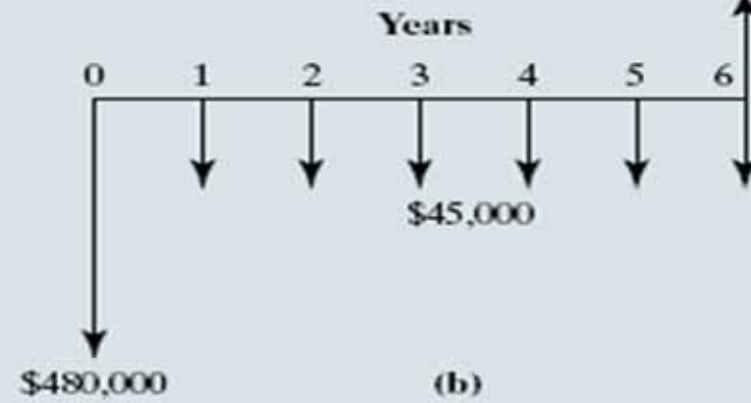
Required Service Period = 2 years

Model A \$50,000



(a)

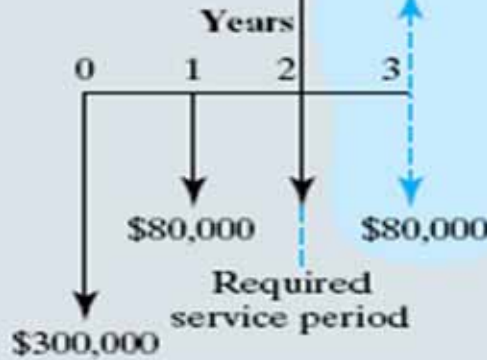
Model B \$60,000



(b)

Estimated salvage value at the end of required service period

Model A \$90,000



PW(15%)A = -\$362,000

Model B \$250,000



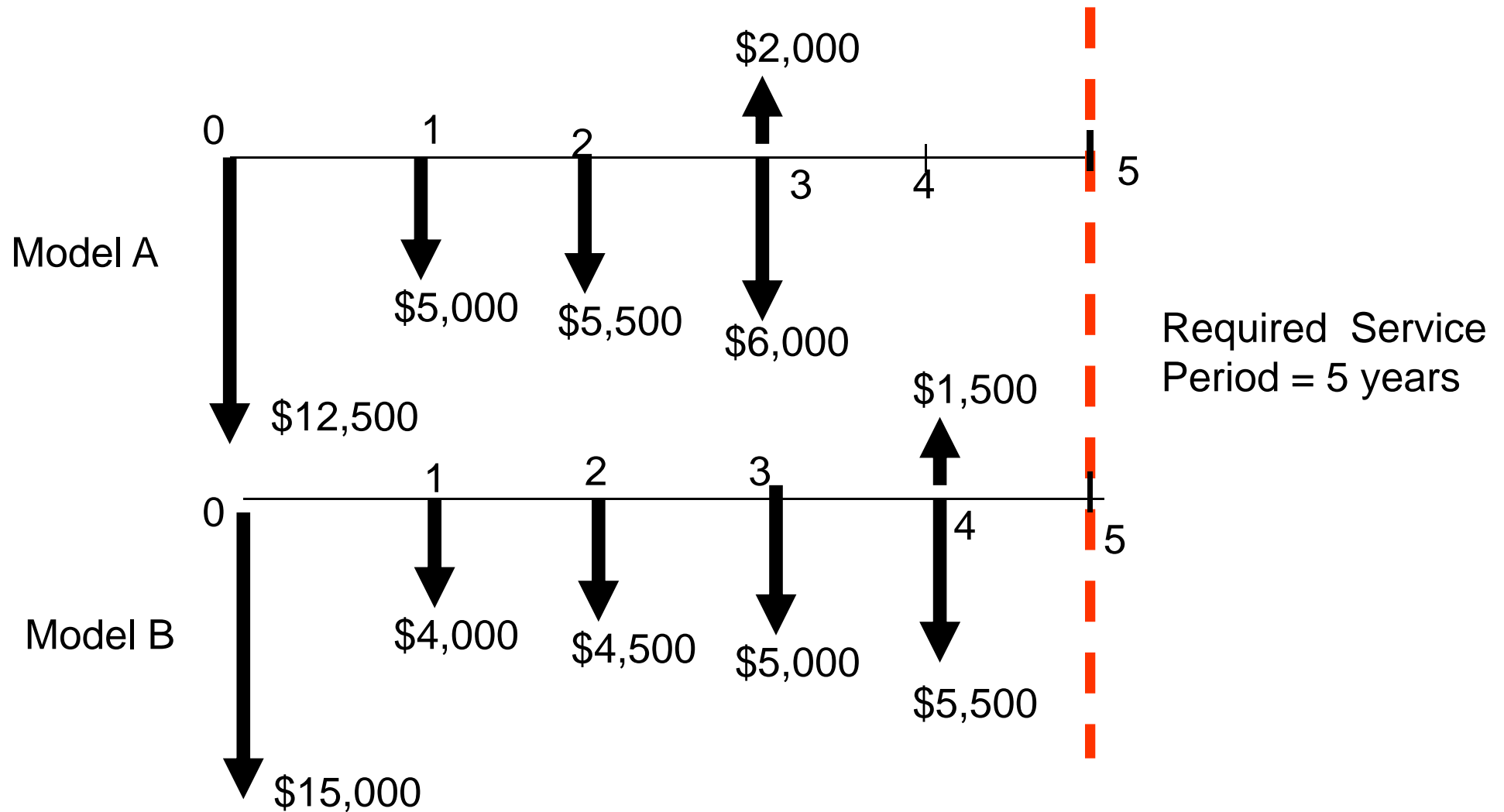
(c)

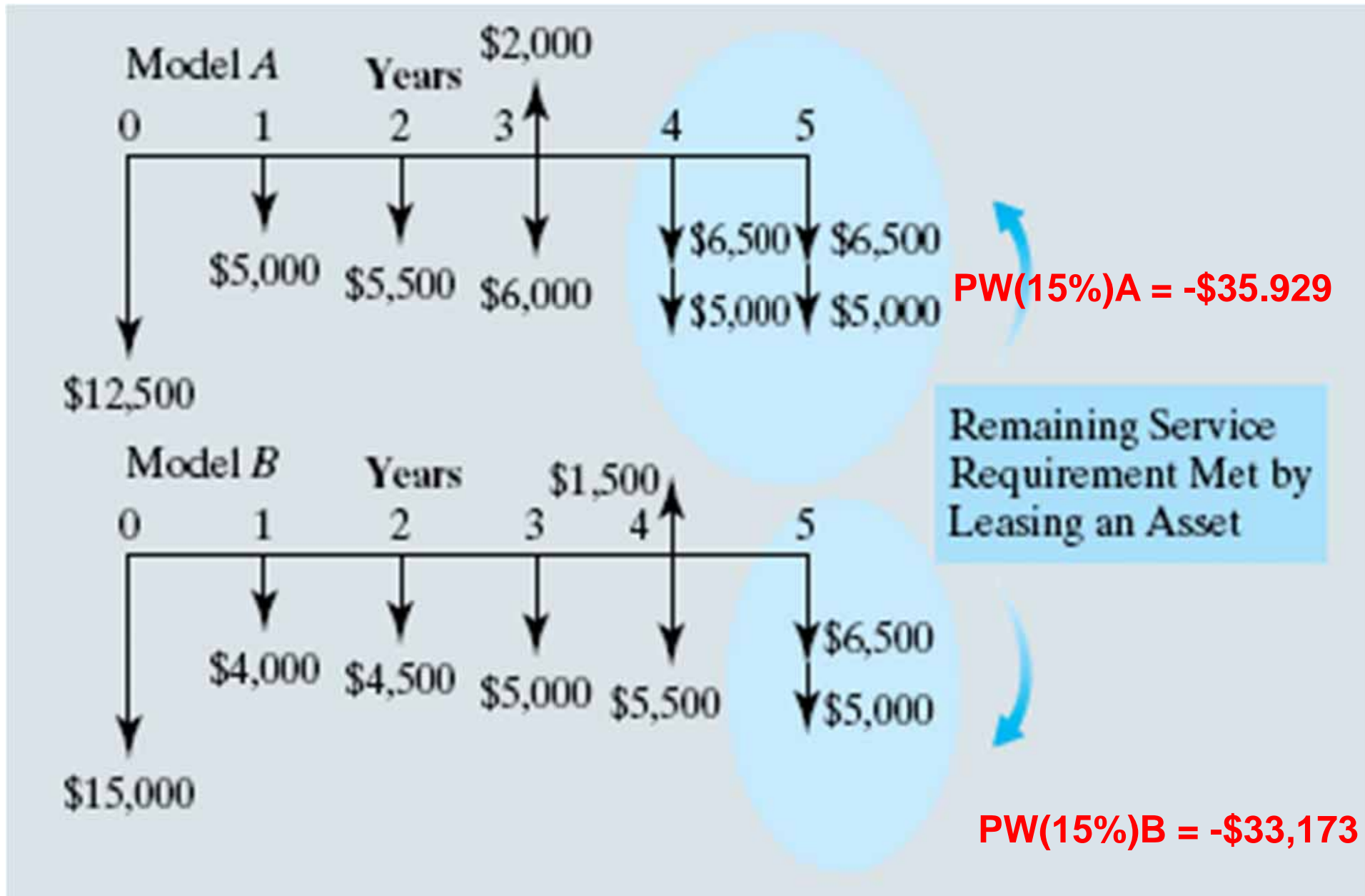
PW(15%)B = -\$364,000

## Case 2: Project lives shorter than the analysis period

- Come up with **replacement projects** that match or exceed the required service period.
- Compute the PW for each project over the required service period

# Example: Comparison for Service Projects with Unequal Lives when the required service period is longer than the individual project life





# Annual Equivalent (AE) Analysis

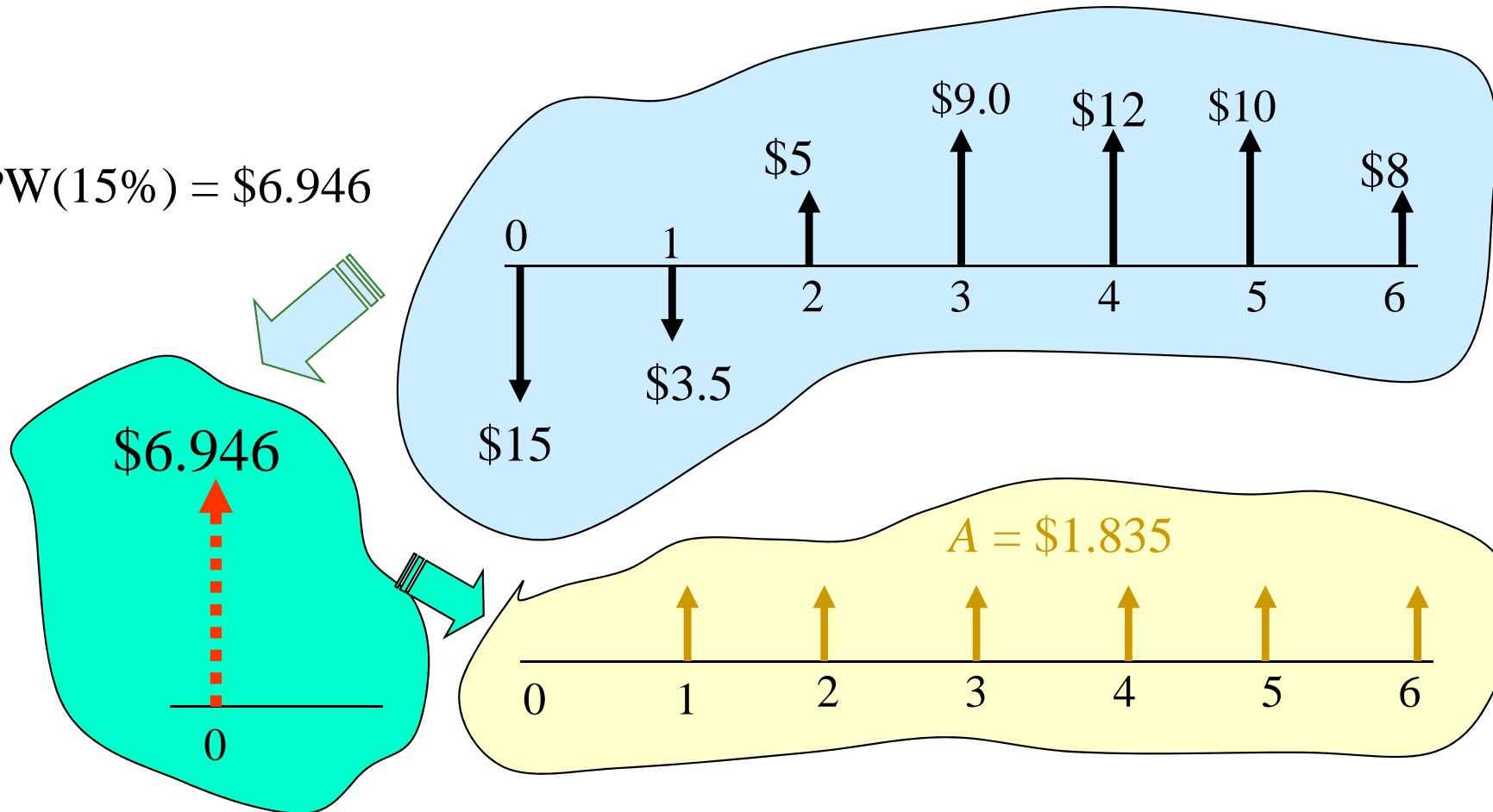
- AE worth criterion provides a basis for measuring investment worth by determining equal payments on annual basis.
- Knowing that any lump-sum cash amount can be converted into a series of equal annual payments
- Find the net present worth of the original series and then multiply this amount by the **capital-recovery factor**:

$$\mathbf{AE (i) = PW (i) \times (A/P, i, N)}$$

- We use this formula to evaluate the investment worth of projects. Therefore, AE criterion provides basis for evaluating a project that is consistent with the PW criterion.

# Example: Computing Equivalent Annual Worth

$$PW(15\%) = \$6.946$$



$$AE(15\%) = \$6.946 (A/P, 15\%, 6) \\ = \$1.835$$

# Benefits of AE Analysis

- In the real world situations, AE analysis is preferred, or demanded, over NPW analysis
- Consider that even corporations issue annual reports and develop yearly budgets.
- For these purposes, a company may find it useful to present the annual cost or benefit of ongoing project rather than its overall cost or benefit. More specifically;
  - **1. When consistency of report formats is desired**
    - Financial managers and engineering managers use annual reports and submit project analysis annual basis that is easy use by other members of the corporation and stock holders.
  - **2. When there is a need to determine unit costs or profits**
    - Projects must be broken into unit cost (or profits) for easy comparison with alternatives.
  - **3. When project lives are unequal**



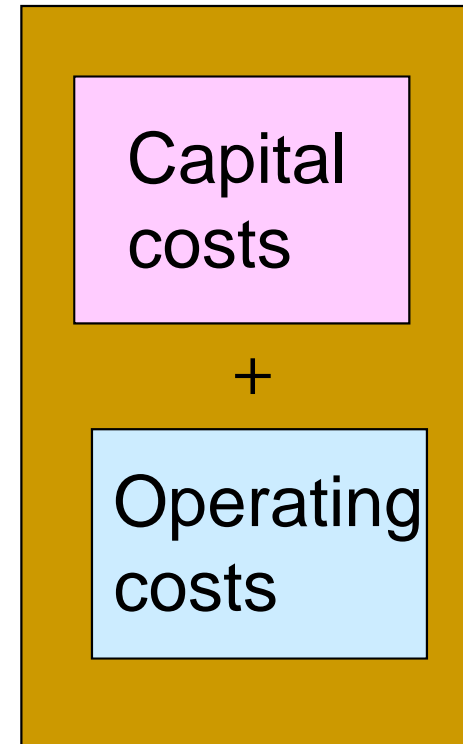
# Fundamental Decision Rules

- Single Project Evaluation:
  - If  $AE(i) > 0$ , accept the investment
  - If  $AE(i) = 0$ , remain indifferent to the investment
  - If  $AE(i) < 0$ , reject the investment
- Comparing Mutually Exclusive Alternatives:
  - Service projects: select the alternative with the minimum annual equivalent cost (AEC)
  - Revenue projects: select the alternative with the maximum  $AE(i)$

# Annual Equivalent Cost

- When only costs are involved, the AE method is called the **annual equivalent cost method**.
- Revenues must cover two kinds of costs: **Operating costs** and **capital costs**.

Annual Equivalent Costs

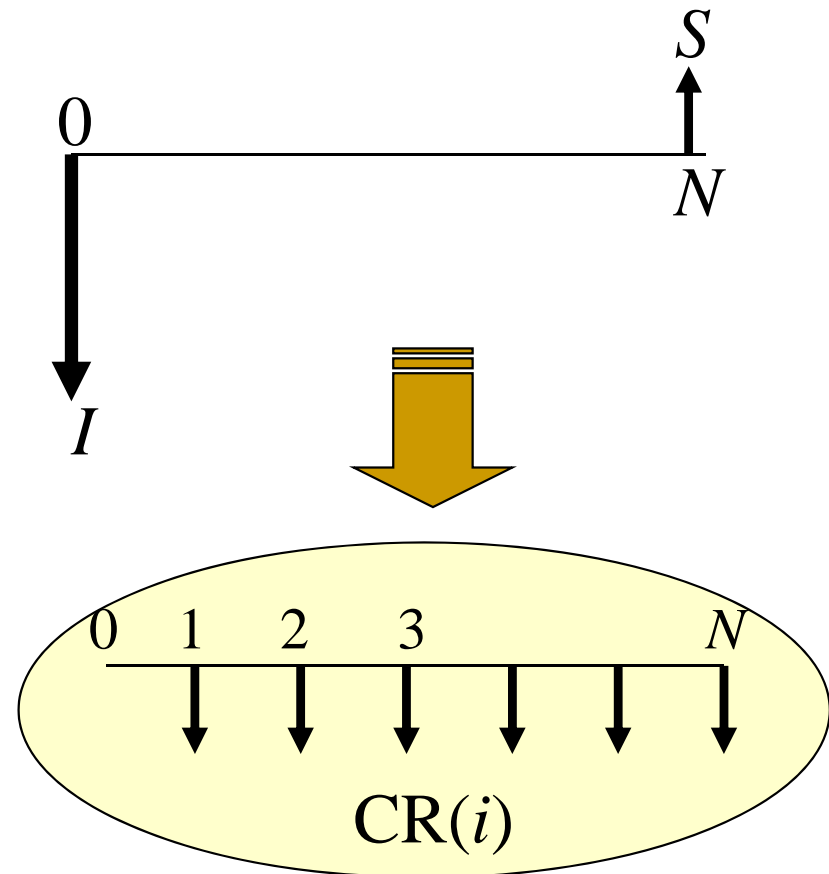


- **Operating costs** are incurred by operation of physical plant or equipment needed to provide service; examples include the costs of items; labor, & raw materials.
- **Capital recovery costs** ( or **ownership costs**) are incurred by purchasing assets to be used in production and service.
- Normally, Capital costs are nonrecurring (one time costs), where as operating costs recur as long as an asset is owned.
- Annual equivalent of a capital cost is given special name: **Capital Recovery cost**, designated **CR (i)**.

# Capital (Ownership) Costs

- **Definition:** Owning equipment is associated with two transactions - (1) its **initial cost ( $I$ )** and (2) its **salvage value ( $S$ )**.
- **Capital costs:** Taking these items into account, we calculate the **capital costs** as:

$$\begin{aligned} CR(i) &= I(A/P, i, M) - S(A/F, i, M) \\ &= (I - S)(A/P, i, M) + iS \end{aligned}$$



# Capital (Ownership) Costs Associated with Various Vehicles

SEGMENT	BEST MODELS	ASKING PRICE	PRICE AFTER 3 YEARS
Compact car	<b>Mini Cooper</b>	<b>\$19,800</b>	<b>\$12,078</b>
Midsize car	Volkswagen Passat	\$28,872	\$15,013
Sports car	Porsche 911	\$87,500	\$48,125
Near luxury car	BMW 3 Series	\$39,257	\$20,806
Luxury car	Mercedes CLK	\$51,275	\$30,765
Minivan	Honda Odyssey	\$26,876	\$15,051
Subcompact SUV	Honda CR-V	\$20,540	\$10,681
Compact SUV	Acura MDX	\$37,500	\$21,375
Full size SUV	Toyota Sequoia	\$37,842	\$18,921
Compact truck	Toyota Tacoma	\$21,200	\$10,812
Full size truck	Toyota Tundra	\$25,653	\$13,083

# Example - Capital Cost Calculation for Mini Cooper

- **Given:**

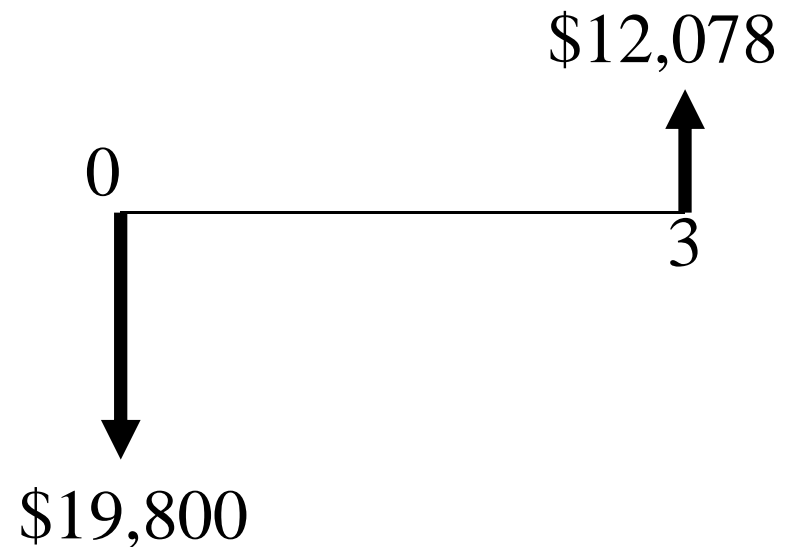
$$I = \$19,800$$

$$N = 3 \text{ years}$$

$$S = \$12,078$$

$$i = 6\%$$

- **Find: CR(6%)**



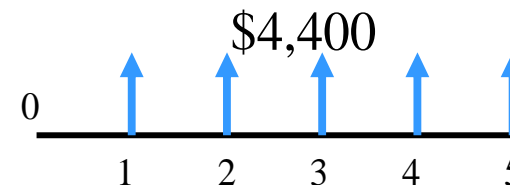
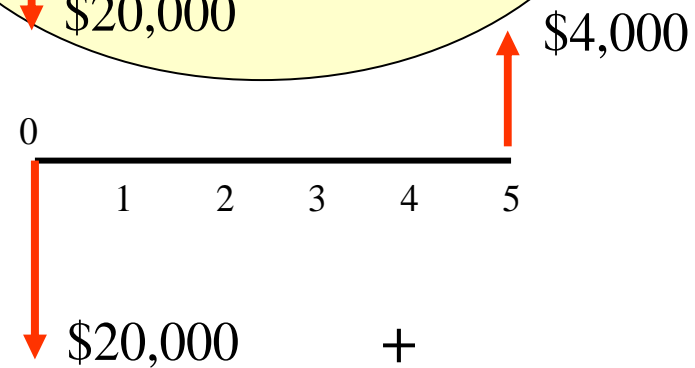
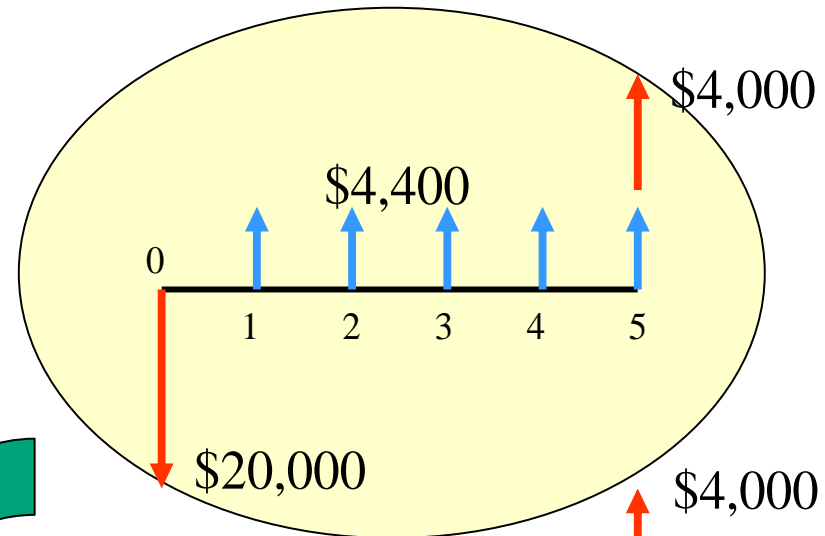
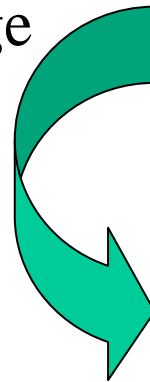
$$CR(i) = (I - S)(A/P, i, N) + iS$$

$$\begin{aligned} CR(6\%) &= (\$19,800 - \$12,078)(A/P, 6\%, 3) \\ &\quad + (0.06)\$12,078 \\ &= \$3,613.55 \end{aligned}$$

# Example

## Justifying an investment based on AE Method

- **Given:**  $I = \$20,000$ ,  $S = \$4,000$ ,  $N = 5$  years,  $i = 10\%$
- **Find:** see if an annual revenue of  $\$4,400$  is large enough to cover the capital costs.
- **Solution:**  
 $CR(10\%) = \$4,620.76$
- **Conclusion:** Need an additional annual revenue in the amount of  $\$220.76$ .

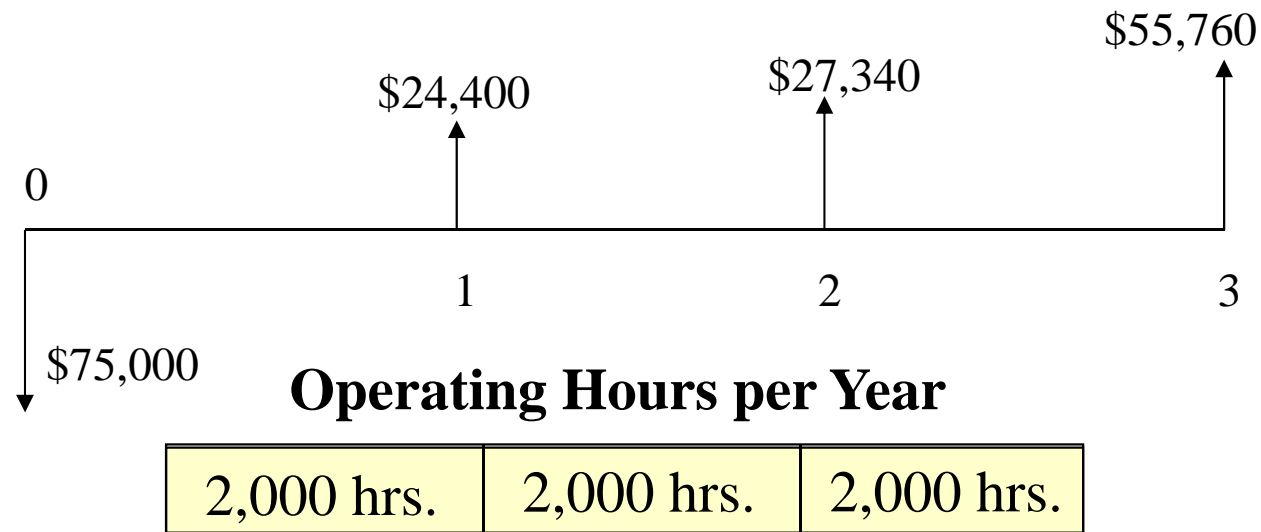


# Annual Worth Analysis

- Where to Apply the AE Analysis
  - Unit cost (or profit) calculation
  - Outsourcing (Make-Buy) Decision
  - Pricing the Use of an Asset
- Unit Cost (Profit) Calculation
  - Step 1: Determine the **number of units (annual volume)** to be produced (or serviced) each year over the life of the asset
  - Step 2: Determine the **annual equivalent cost (or worth)** by owning and operating the asset
  - Step 3: Divide the equivalent cost (worth) by the annual volume



# Example: Equivalent Worth per Unit of Time



Step 1: Determine the annual volume - 2,000 hours per year

Step 2: Obtain the equivalent annual worth

$$PW (15\%) = \$3553$$

$$AE (15\%) = \$3,553 (A/P, 15\%, 3)$$
$$= \$1,556$$

Step 3: Determine the unit profit (savings per machine hour)

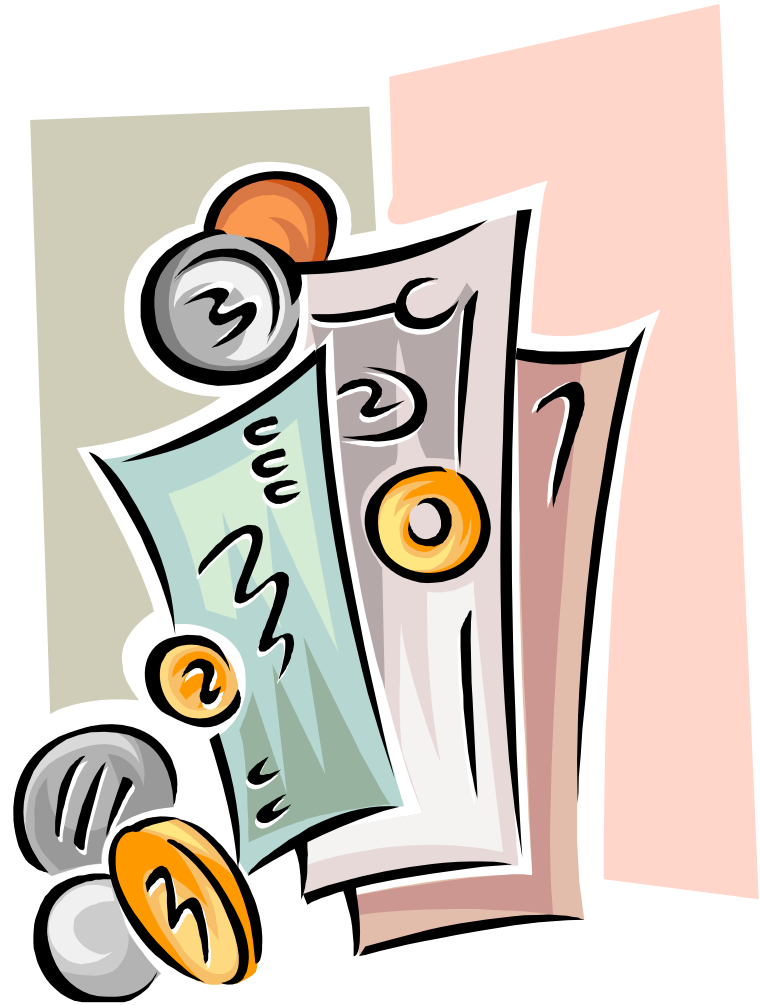
Savings per Machine Hour

$$= \$1,556/2,000$$

$$= \$0.78/\text{hr.}$$

# Rate of Return Analysis

- ❑ Rate of Return (ROR)
- ❑ Methods for Finding ROR
- ❑ Internal Rate of Return (IRR) Criterion
- ❑ Incremental Analysis



# Rate of Return Analysis

## Definition

A relative percentage method which measures the annual rate of return as a percentage of investment over the life of a project.

- Interest earned on your invested capital, or commonly known as internal rate of return (IRR)
- A Simple Example: The interest earned on your savings account is the rate of return on your deposits

# Rate of Return Analysis

- Investopedia ([www.investopedia.com](http://www.investopedia.com)) says:
  - IRRs can also be compared against prevailing rates of return in the securities market. If a firm can't find any projects with IRRs greater than the returns that can be generated in the financial markets, it may simply choose to invest its retained earnings into the market.

# Example: Meaning of Rate of Return: Investing in Wal-Mart Stock

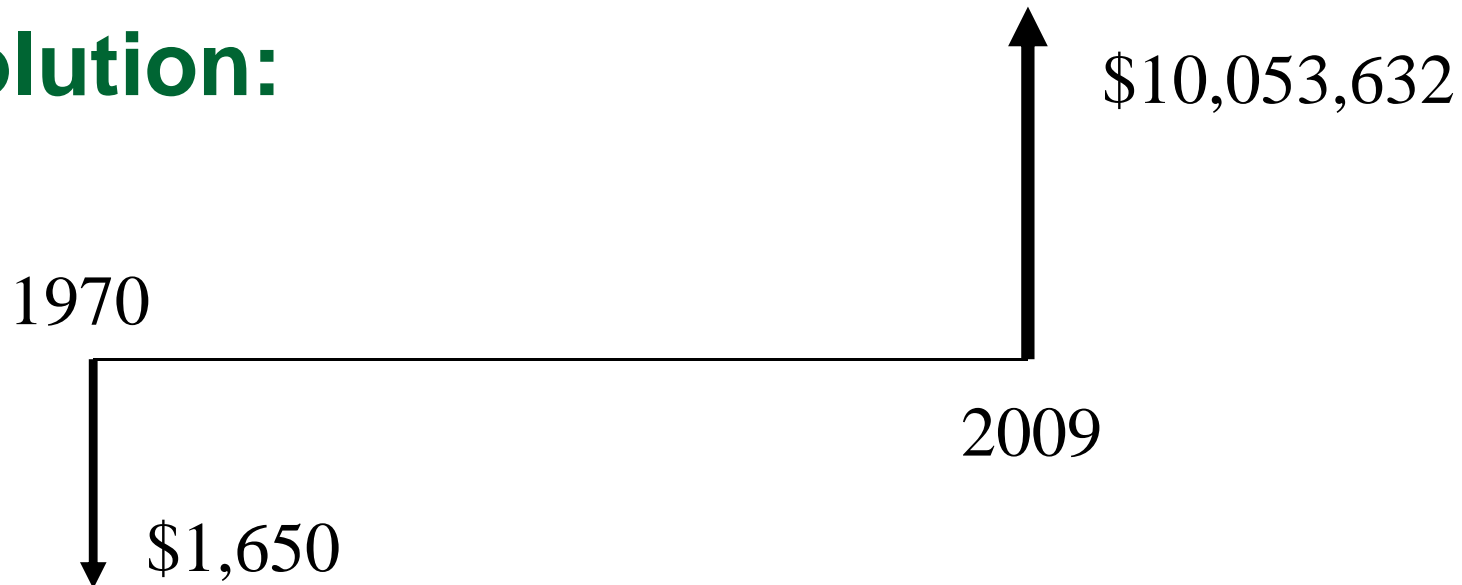
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In 1970, when Wal-Mart Stores, Inc. went public, an investment of 100 shares cost \$1,650. That investment would have been worth \$10,053,632 on September 30, 2009.

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**What is the rate of return on that investment?**

## Solution:



**Given:**  $P = \$1,650$

$F = \$10,053,632$

$N = 29$  years

**Find  $i$ :**

$$F = P (1 + i)^N$$

$$\$ 10,053,632 = \$1,650 (1 + i)^{29}$$

$$i = \underline{25.04\%} \quad \Rightarrow \text{Rate of Return}$$

# Wal-Mart Investment Problem

Suppose that you invested that amount (\$1,650) in a savings account at 6% per year. Then, you could have only \$16,010 on January, 2009.

---

What is the meaning of this 6% interest here?

---

This is your opportunity cost rate or minimum return required if putting money in savings account was the best you can do at that time!

So, in 1970, as long as you earn more than 6% interest in another investment, you will take that investment.

---

Therefore, that 6% is viewed as a minimum attractive rate of return (MARR) (or required rate of return). This is the interest rate used in NPW analysis

---

So, you can apply the following decision rule, to see if the proposed investment is a good one.

$$\text{ROR (25.04\%)} > \text{MARR(6\%)}$$



# Why ROR measure is so popular?

- This project will bring in a **15% rate of return** on investment.
- This project will result in a **net surplus of \$10,000** in NPW.
  - Which statement is easier to understand?

## **Three Definitions of ROR:**

1. Interest Earned on Loan Balance
2. Break-Even Interest Rate
3. Internal Rate of Return

# Return on Investment

## Interest Earned on Loan Balance

**Definition 1:** Rate of return (ROR) is defined as the interest rate earned on the *unpaid (outstanding) balance* of an installment loan.

**Example:** A bank lends \$10,000 and receives annual payment of \$4,021 over 3 years. The bank is said to earn a *return of 10%* on its loan of \$10,000.

# Loan Balance Calculation:

$$A = \$10,000 (A/P, 10\%, 3) \\ = \$4,021$$

Year	Unpaid balance at beg. of year	Return on unpaid balance (10%)	Payment received	Unpaid balance at the end of year
0	-\$10,000			-\$10,000
1	-\$10,000	-\$1,000	+\$4,021	-\$6,979
2	-\$6,979	-\$698	+\$4,021	-\$3,656
3	-\$3,656	-\$366	+\$4,021	0

A return of 10% on the amount **still outstanding** at the beginning of each year

# Return on Investment

## Break-Even Interest Rate

**Definition 2:** Rate of return (ROR) is the **break-even interest rate**,  $i^*$ , which equates the present worth of a project's cash outflows to the present worth of its cash inflows.

### Mathematical Relation:

$$\begin{aligned}PW(i^*) &= PW(i^*)_{\text{cash inflows}} - PW(i^*)_{\text{cash outflows}} \\ &= 0\end{aligned}$$

### Example:

$$PW(10\%) = -10,000 + \$4,021(P/A, 10\%, 3) = 0$$

# Return on Invested Capital (RIC)

## Internal Rate of Return

**Definition 3:** The **internal rate of return** (IRR) is the interest rate earned on the **unrecovered project balance** of the investment such that, when the project terminates, the unrecovered project balance will be zero.

**Example:** A company invests \$10,000 in a computer system which results in equivalent annual labor savings of \$4,021 over 3 years. The company is said to earn a **return of 10%** on its investment of \$10,000.

# Project Balance Calculation:

	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>
Beginning project balance		-\$10,000	-\$6,979	-\$3,656
Return on invested capital		-\$1,000	-\$697	-\$365
Payment received	-\$10,000	+\$4,021	+\$4,021	+\$4,021
Ending project balance	-\$10,000	-\$6,979	-\$3,656	0


The firm earns a 10% rate of return on funds that remain **internally invested in the project**. Since the return is **internal** to the project, we call it **internal rate of return**.

# Methods for Finding Rate of Return


- **Types of Investment (cash flow) Classification**
    - Simple Investment
    - Non-simple Investment
  - Once we identified the type of investment cash flow, there are several ways available to determine its rate of return.
- 
- **Computational Methods**
    - Direct Solution Method
    - Trial-and-Error Method
    - Computer Solution Method
-

# Investment Classification

## Simple Investment

- **Definition:** Initial cash flows are negative, and only one sign change occurs in the net cash flows series.
- **Example:** -\$100, 250, \$300  (-, +, +)
- **ROR:** A unique ROR
- If the initial flows are positive and one sign change occurs referred to simple-borrowing.

## Non-simple Investment

- **Definition:** Initial cash flows are negative, but more than one sign changes in the remaining cash flow series.
- **Example:** -\$100, 300, -\$120  (-, +, -)
- **ROR:** A possibility of multiple RORs

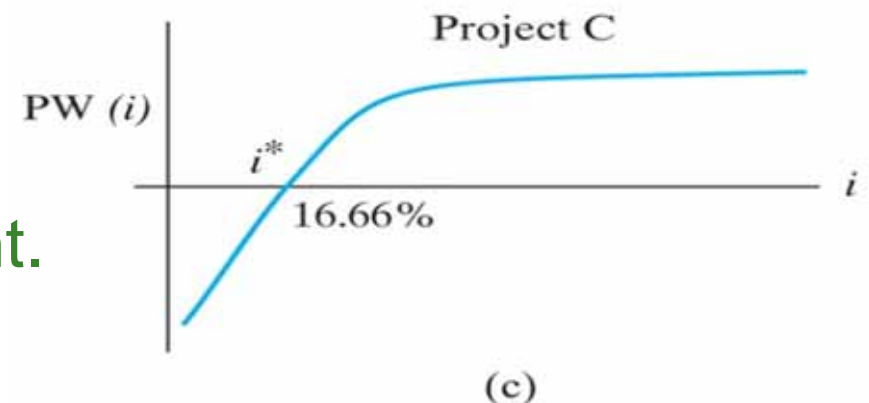
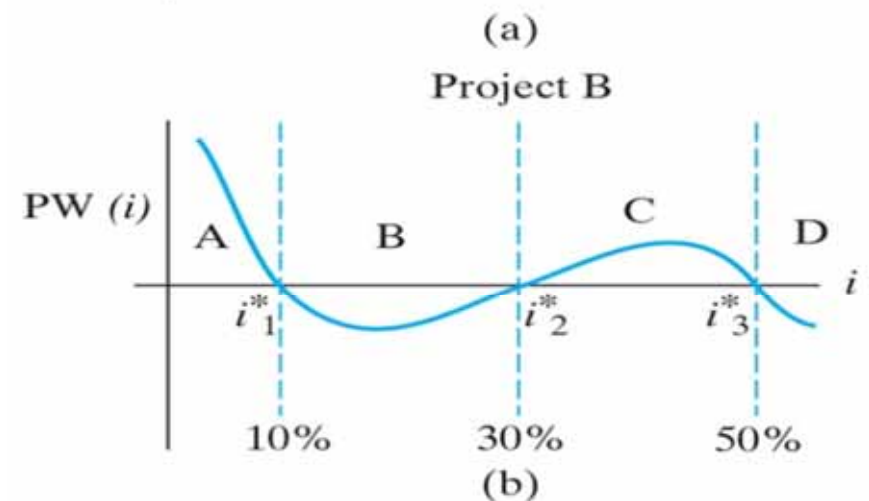
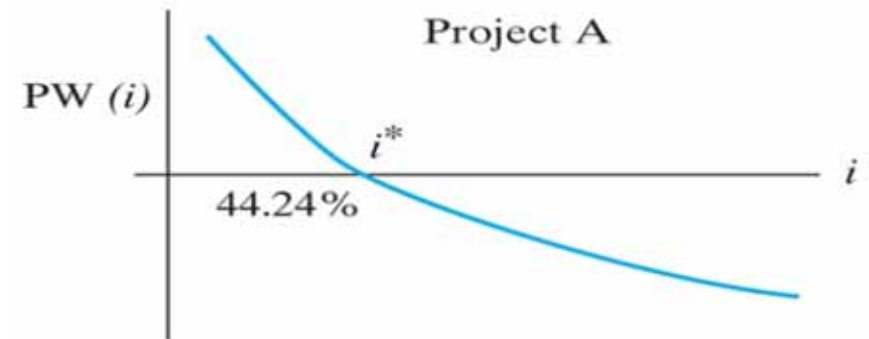


# Investment Classification: Example

Net Cash Flow

Period (N)	Project A	Project B	Project C
0	-\$1,000	-\$1,000	+\$1,000
1	-500	3,900	-450
2	800	-5,030	-450
3	1,500	2,145	-450
4	2,000		

Project A is a simple investment.  
 Project B is a non-simple investment.  
 Project C is a simple borrowing.



# Finding Rate of Return: Example

## Computational Methods

- Using Excel's Financial Command
- Direct Solution Method
- Trial-and-Error Method (works only for simple investment)

Excel command to find the rate of return:

**=IRR(cell range, guess)**

e.g., =IRR(C0:C7, 10%)

# Finding Rate of Return: Example

## Computational Methods

	Direct Solution	Direct Solution	Trial & Error Method	Computer Solution Method
	Log	Quadratic		
$n$	Project A	Project B	Project C	Project D
0	-\$1,000	-\$2,000	-\$75,000	-\$10,000
1	0	1,300	24,400	20,000
2	0	1,500	27,340	20,000
3	0		55,760	25,000
4	1,500			

# Finding Rate of Return: Example

## Direct Solution Methods

### • Project A

$$\$1,000 = \$1,500(P / F, i, 4)$$

$$\$1,000 = \$1,500(1 + i)^{-4}$$

$$0.6667 = (1 + i)^{-4}$$

$$\frac{\ln 0.6667}{-4} = \ln(1 + i)$$

$$0.101365 = \ln(1 + i)$$

$$e^{0.101365} = 1 + i$$

$$i = e^{0.101365} - 1$$

$$= 10.67\%$$

### • Project B

$$PW(i) = -\$2,000 + \frac{\$1,300}{(1+i)} + \frac{\$1,500}{(1+i)^2} = 0$$

Let  $x = \frac{1}{1+i}$ , then

$$PW(i) = -2,000 + 1,300x + 1,500x^2$$

Solve for  $x$ :

$$x = 0.8 \text{ or } -1.667$$

Solving for  $i$  yields

$$0.8 = \frac{1}{1+i} \rightarrow i = 25\%, \quad -1.667 = \frac{1}{1+i} \rightarrow i = -160\%$$

Since  $-100\% < i < \infty$ , the project's  $i^* = 25\%$ .

# Trial and Error Method – Project C

- Step 1: Guess an interest rate, say,  $i = 15\%$
- Step 2: Compute  $PW(i)$  at the guessed  $i$  value.

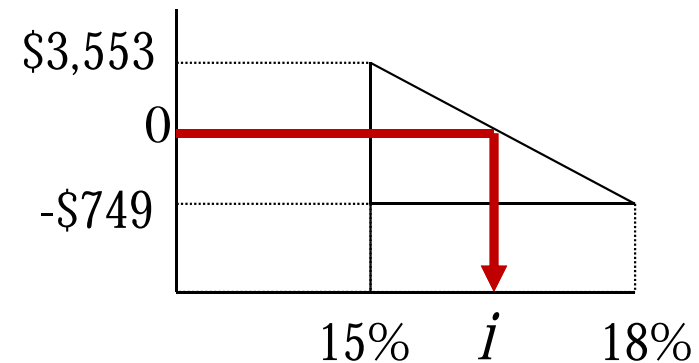
$$PW(15\%) = \$3,553$$

- Step 3: If  $PW(i) > 0$ , then increase  $i$ . If  $PW(i) < 0$ , then decrease  $i$ .

$$PW(18\%) = -\$749$$

Note: This method works only for finding  $i^*$  for simple investments.

- Step 4: If you bracket the solution, you use a linear interpolation to approximate the solution



$$i = 15\% + 3\% \left[ \frac{3,553}{3,553 + 749} \right]$$
$$= 17.45\%$$

# Basic Decision Rule:

If  $ROR > MARR$ , Accept

This rule does not work for a situation where an investment has multiple rates of return

# Comparing Mutually Exclusive Alternatives Based on IRR

**Issue:** Can we rank the mutually exclusive projects by the magnitude of its IRR?

<u><i>n</i></u>	<u>A1</u>		<u>A2</u>
<b>0</b>	-\$1,000		-\$5,000
<b>1</b>	\$2,000		\$7,000
<hr/>			
<b>IRR</b>	100%	>	40%
<b>PW (10%)</b>	\$818	<	\$1,364
<hr/>			

# Who Got More Pay Raise?



Billy

10%



Nancy

5%



# Can't Compare without Knowing Their Base Salaries

	Billy	Nancy
Base Salary	\$50,000	\$200,000
Pay Raise (%)	10%	5%
Pay Raise (\$)	\$5,000	\$10,000

For the same reason, we can't compare mutually exclusive projects based on the magnitude of its IRR. We need to know the size of investment and its timing of when to occur.

# Incremental Investment Analysis

**At Issue:** Can we justify the higher cost investment, say A2?

$n$	Project A1	Project A2	Incremental Investment (A2 – A1)
0	-\$1,000	-\$5,000	-\$4,000
1	\$2,000	\$7,000	\$5,000
ROR	100%	40%	25%
PW(10%)	\$818	\$1,364	\$546

➤ Assuming a MARR of 10%, you can always earn that rate from other investment source, i.e., \$4,400 at the end of one year for \$4,000 investment.

➤ By investing the additional \$4,000 in A2, you would make additional \$5,000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment in A2 is justified.



# Incremental Analysis (Procedure)

**Step 1:** Compute the cash flow for the difference between the projects (A,B) by subtracting the cash flow of the **lower** investment cost project (A) from that of the **higher** investment cost project (B).

**Step 2:** Compute the IRR on this incremental investment ( $IRR_{B-A}$ ).

**Step 3:** Accept the investment **B** if and only if

$$IRR_{B-A} > MARR$$

NOTE: Make sure that both  $IRR_A$  and  $IRR_B$  are greater than MARR.

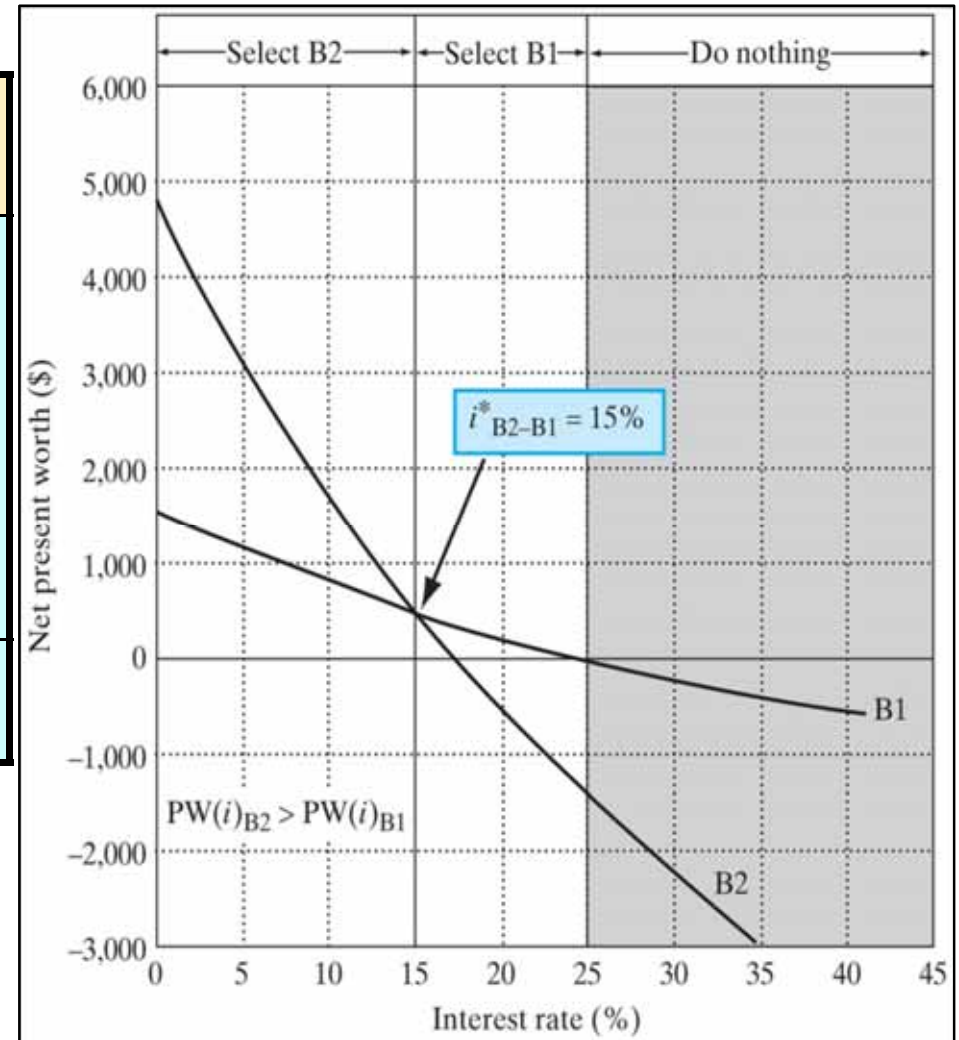
# Example: Incremental Rate of Return: Two Alternatives

- Project Cash Flows:

$n$	B1	B2	B2 - B1
0	-\$3,000	-\$12,000	-\$9,000
1	1,350	4,200	2,850
2	1,800	6,225	4,425
3	1,500	6,330	4,830
IRR	25%	17.43%	15%

Given MARR = 10%, which project is a better choice?

Conclusion: Since  $IRR_{B2-B1} = 15\% > 10\%$ , and also  $IRR_{B2} > 10\%$ , select B2.



# Example: IRR on Increment Investment: Three Alternatives

- Given: MARR = 15%

$n$	D1	D2	D3
0	-\$2,000	-\$1,000	-\$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000
IRR	34.37%	40.76%	24.81%

**Step 1:** Examine the IRR for each project to eliminate any project that fails to meet the MARR.

**Step 2:** Compare D1 and D2 in pairs.  
 $IRR_{D1-D2} = 27.61\% > 15\%$ ,  
 so select **D1**. D1 becomes the current best.

**Step 3:** Compare D1 and D3.  
 $IRR_{D3-D1} = 8.8\% < 15\%$ ,  
 so select **D1** again.

Here, we conclude that **D1** is the best Alternative.

# Example: Incremental Analysis for Cost-Only Projects

Items	CMS Option	FMS Option
Investment	\$ 4,500,000	\$ 12,500,000
Total annual operating costs	\$ 7,412,920	\$ 5,504,100
Net salvage value	\$ 500,000	\$ 1,000,000

- The firm's MARR is 15%. Which alternative would be a better choice, based on the IRR criterion?
- **Discussion:** Since we can assume that both manufacturing systems would provide the same level of revenues over the analysis period, we can compare these alternatives based on cost only. (these systems are service projects).
- Although we can not compute the IRR for each option without knowing the revenue figures, we can still calculate the IRR on incremental cash flows.
- Since the FMS option requires a higher initial investment than that of the CMS, the incremental cash flow is the difference (FMS – CMS)

## Example: Incremental Analysis for Cost-Only Projects (cost are itemized)

Items	CMS Option	FMS Option
Annual O&M costs:		
Annual labor cost	\$1,169,600	\$707,200
Annual material cost	832,320	598,400
Annual overhead cost	3,150,000	1,950,000
Annual tooling cost	470,000	300,000
Annual inventory cost	141,000	31,500
Annual income taxes	1,650,000	1,917,000
Total annual operating costs	<u>\$7,412,920</u>	<u>\$5,504,100</u>
Investment	\$4,500,000	\$12,500,000
Net salvage value	\$500,000	\$1,000,000

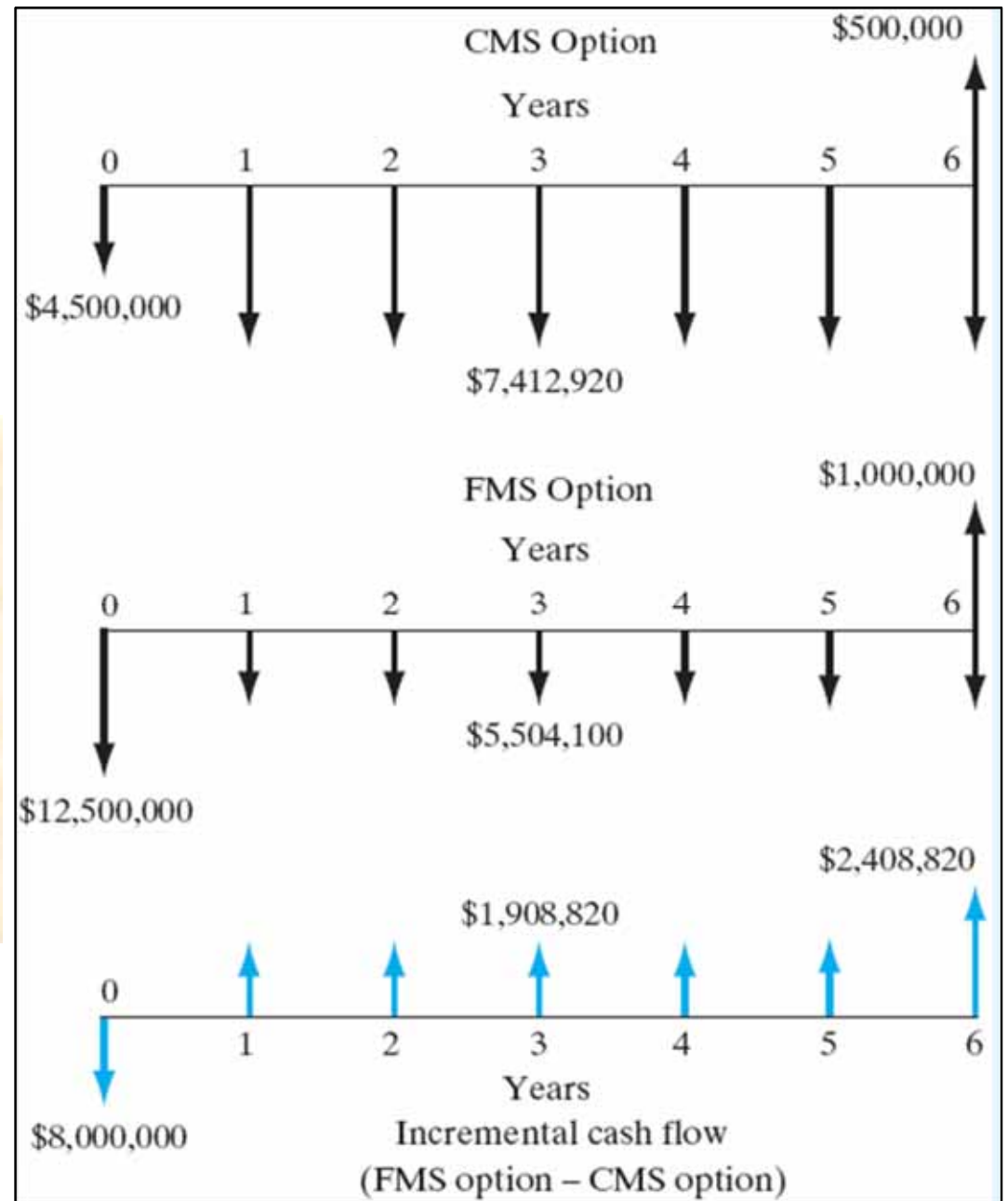
## Example: Incremental Cash Flow (FMS – CMS)

$n$	CMS Option	FMS Option	Incremental (FMS-CMS)
0	-\$4,500,000	-\$12,500,000	-\$8,000,000
1	-7,412,920	-5,504,100	1,908,820
2	-7,412,920	-5,504,100	1,908,820
3	-7,412,920	-5,504,100	1,908,820
4	-7,412,920	-5,504,100	1,908,820
5	-7,412,920	-5,504,100	1,908,820
6	-7,412,920	-5,504,100	\$2,408,820
Salvage	+ \$500,000	+ \$1,000,000	



# Solution:

$$\begin{aligned}
 PW(i)_{FMS-CMS} &= -\$8,000,000 \\
 &\quad +\$1,908,820(P/A, i, 5) \\
 &\quad +\$2,408,820(P/F, i, 6) \\
 &= 0 \\
 IRR_{FMS-CMS} &= 12.43\% < 15\%, \\
 &\text{select CMS.}
 \end{aligned}$$



- Although the FMS would provide an incremental annual savings of \$1,908,820 in operating costs, the savings do not justify the incremental investment of \$8,000,000.

### **COMMENTS:**

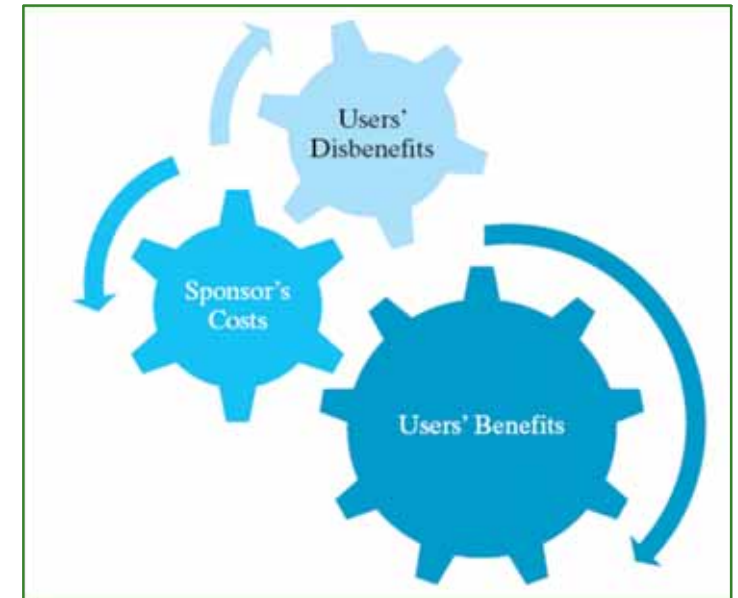
- Note that the CMS option was marginally preferred to the FMS option.
- However, there are dangers in relying solely on the easily quantified savings in input factors – such as labor, energy, and materials – from FMS and in not considering gains from improved manufacturing performance that are more difficult and subjective to quantify.
- Factors such as improved product quality, increased manufacturing flexibility (rapid response to customer demand), reduced inventory levels, and increased capacity for product innovation are frequently ignored in financial analysis because we have inadequate means for quantifying benefits.
- If these intangible benefits were considered, as they ought to be, however, the FMS option could come out better than the CMS option.

# Benefit-Cost Analysis

- **Benefit-cost (BC) analysis** is a decision making tool commonly used to systematically develop useful information about the desirable and undesirable effects of public projects
- Benefits of a nonmonetary nature need to be quantified in dollar terms as much as possible and factored into the analysis
- A broad range of project users distinct from the sponsor can and should be considered -- benefits and disbenefits to all these users can and should be taken into account
- In the Benefits-cost analysis determining social benefits of a public activity is more important than costs

# Framework of Benefit-Cost Analysis

- ❑ **Step 1:** Identifying all the users and sponsors of the project.
- ❑ **Step 2:** Identifying all the benefits and disbenefits of the project.
- ❑ **Step 3:** Quantifying all benefits and disbenefits in dollars or some other unit of measure.
- ❑ **Step 4:** Selecting an appropriate interest rate at which to discount benefits and costs in future to a present value.



Accept the project if the equivalent users' benefits exceed the equivalent sponsors' costs.

# Benefit-Cost Ratios

- Alternative way to express the value of a public project is to compare the users' benefits (**B**) to sponsors' cost (**C**) by taking the ratio **B/C**.
- Define the benefit-cost (**B/C**) ratio, and explain the relationship between the conventional NPW criterion and the B/C ratio.

$$\text{Benefit - Cost Ratio} = \frac{\text{Equivalent Users' Net Benefits}}{\text{Equivalent Sponsor's Net Cost}}$$

If this **BC** ratio exceeds **1**, the project can be justified

# Definition of Benefit-Cost Ratio

$$B = \sum_{n=0}^N b_n (1+i)^{-n}$$

$$C = \sum_{n=0}^N c_n (1+i)^{-n}$$

$b_n$  = Benefit at the end of period  $n$ ,  $b_n \geq 0$

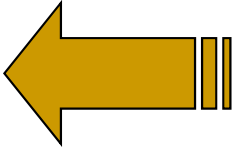
$c_n$  = Expense at the end of period  $n$ ,  $c_n \geq 0$

$A_n = b_n - c_n$

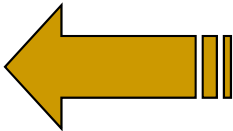
$N$  = Project life

$i$  = Sponsor's interest rate (discount rate)

# Breakdown of the Sponsor's Cost

$$I = \sum_{n=0}^K c_n (1+i)^{-n}$$


Equivalent capital investment at  $n = 0$

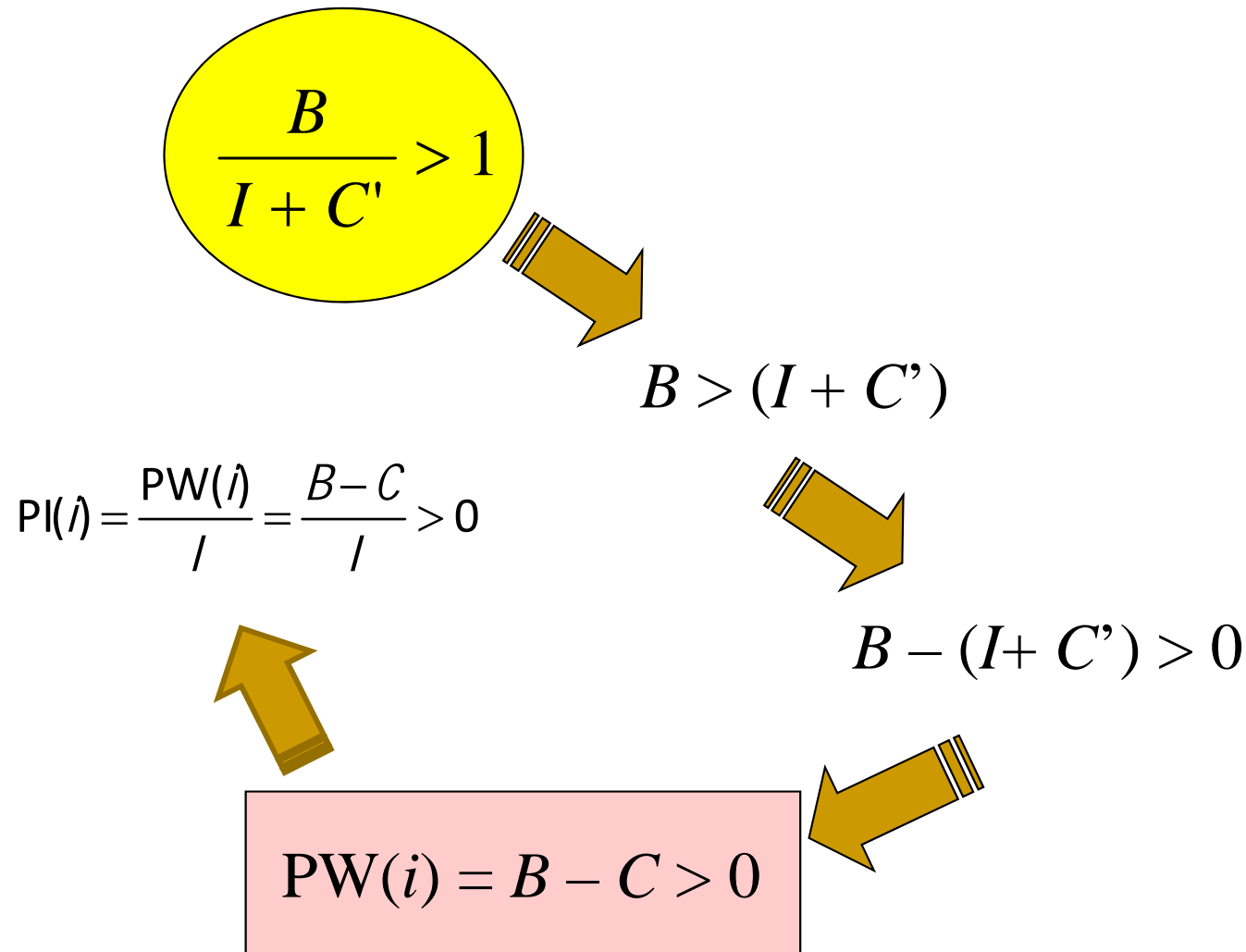
$$C' = \sum_{n=K+1}^N c_n (1+i)^{-n}$$


Equivalent O&M costs at  $n = 0$

$$BC(i) = \frac{B}{C} = \frac{B}{I + C'}, \quad I + C' > 0$$

- The sponsor's cost (  $C$  ) consist of the capital expenditure (  $I$  ) and the equivalent annual operating and maintenance costs (  $C'$  ) accumulated in each successive period.
- Let's assume series of initial investment required during the first  $K$  periods, while annual operating and maintenance costs accumulate in each period.

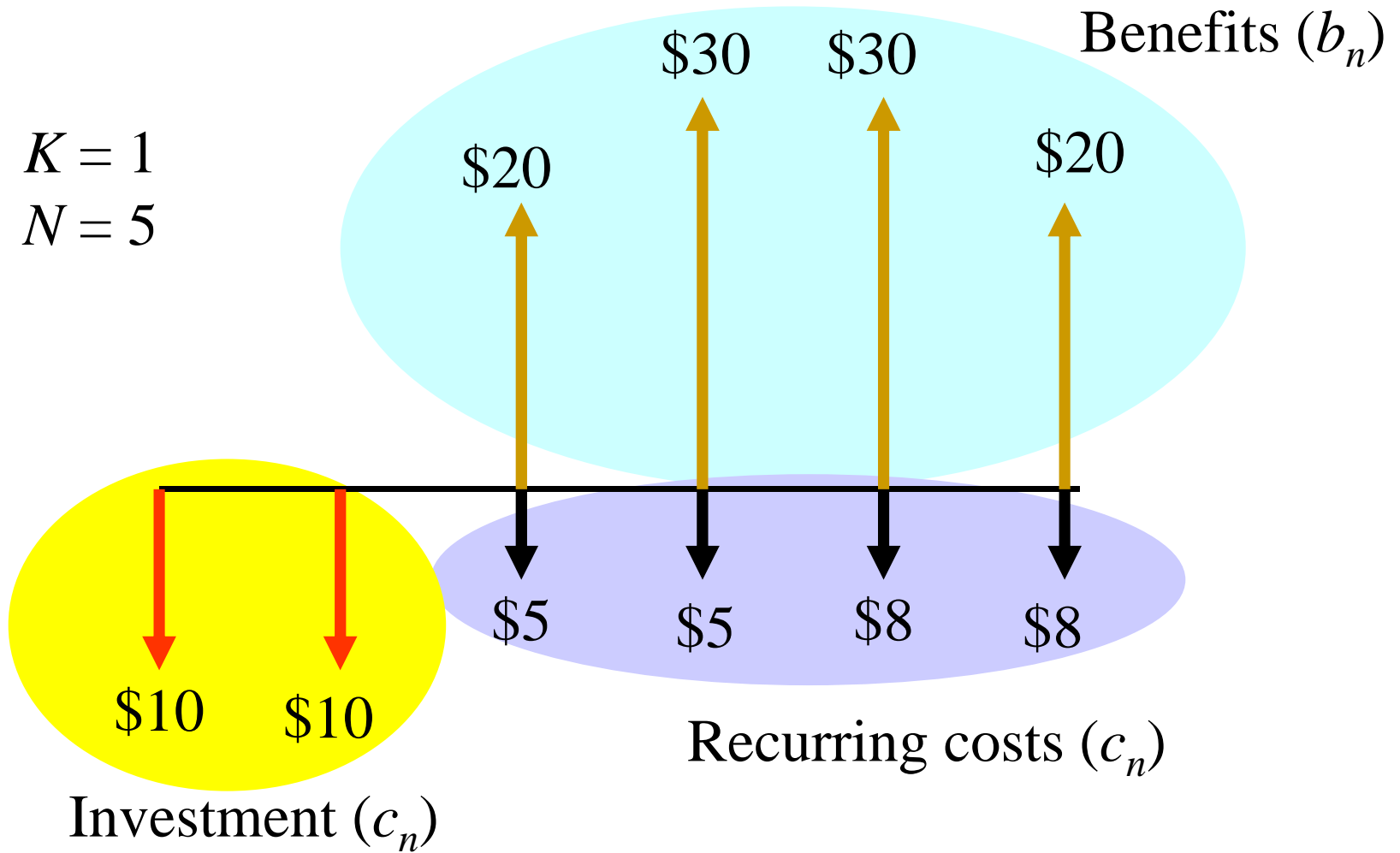
# Relationship between B/C Ratio, NPW, and PI (profitability index)



PI (profitability index) = profit investment ratio (PIR)  
= (Present value of future cash flows)/(Initial investment)



# Example: BC Analysis



# Solution:

$$\begin{aligned} B &= \$20(P / F, 10\%, 2) + \$30(P / F, 1\%, 3) \\ &\quad + \$30(P / F, 10\%, 4) + \$20(P / F, 10\%, 5) \\ &= \$71.98 \end{aligned}$$

$$\begin{aligned} C &= \$10 + \$10(P / F, 10\%, 1) + \$5(P / F, 10\%, 2) + \$5(P / F, 10\%, 3) \\ &\quad + \$8(P / F, 10\%, 4) + \$8(P / F, 10\%, 5) \\ &= \$37.41 \end{aligned}$$

$$\begin{aligned} I &= \$10 + \$10(P / F, 10\%, 1) \\ &= \$19.09 \end{aligned}$$

$$\begin{aligned} C' &= C - I \\ &= \$18.3 \end{aligned}$$

$$BC(10\%) = \frac{71.98}{\$19.09 + \$18.32} = 1.92 > 1, \text{ Accept the project.}$$