BBSE3009 Project Management and Engineering Economics

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Engineering economics analysis

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Nominal and Effective Interest Rates

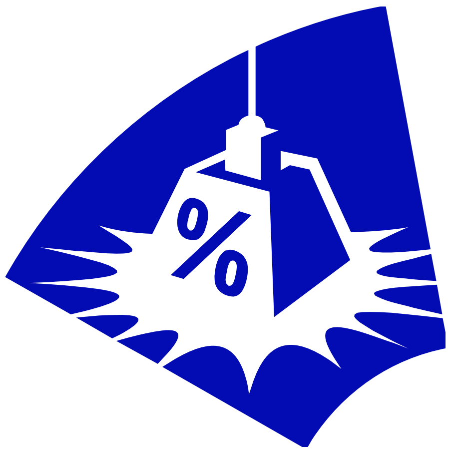
Main Focus:

1. If payments occur more frequently than annual, how do you calculate economic equivalence?
2. If interest period is other than annual, how do you calculate economic equivalence?
3. How are commercial loans structured?
4. How should you manage your debt?

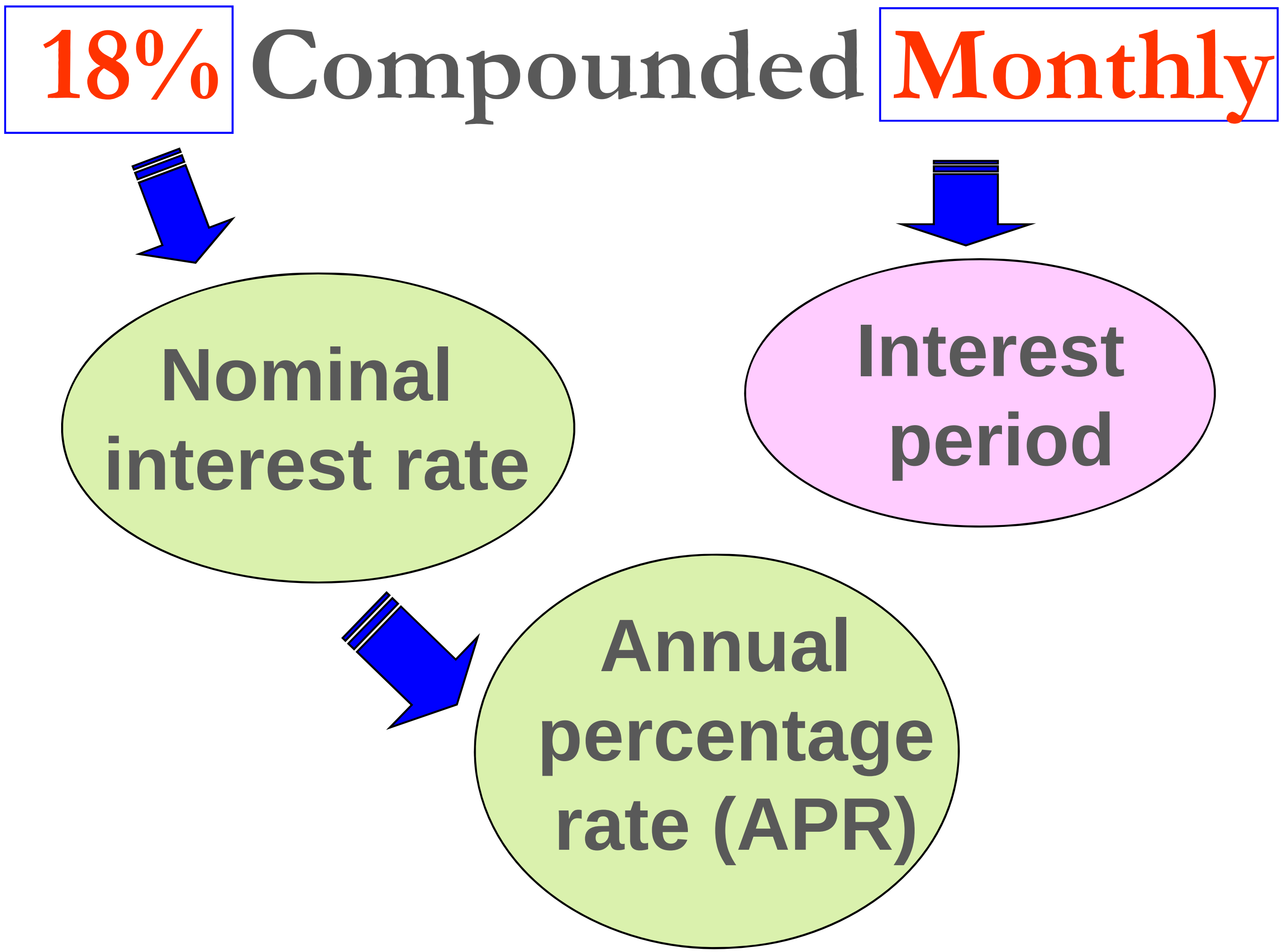
Nominal and Effective Interest Rates

Nominal Interest Rate: 名義利率
Interest rate quoted based on an annual period

Effective Interest Rate: 實質利率
Actual interest earned or paid in a year or some other time period



Financial Jargon



18% Compounded Monthly

- What It Really Means?
  - Interest rate per month (i) = 18% / 12 = 1.5%
  - Number of interest periods per year (N) = 12

- In other words,
  - Bank will charge 1.5% interest each month on your unpaid balance, if you borrowed money
  - You will earn 1.5% interest each month on your remaining balance, if you deposited money

18% Compounded Monthly

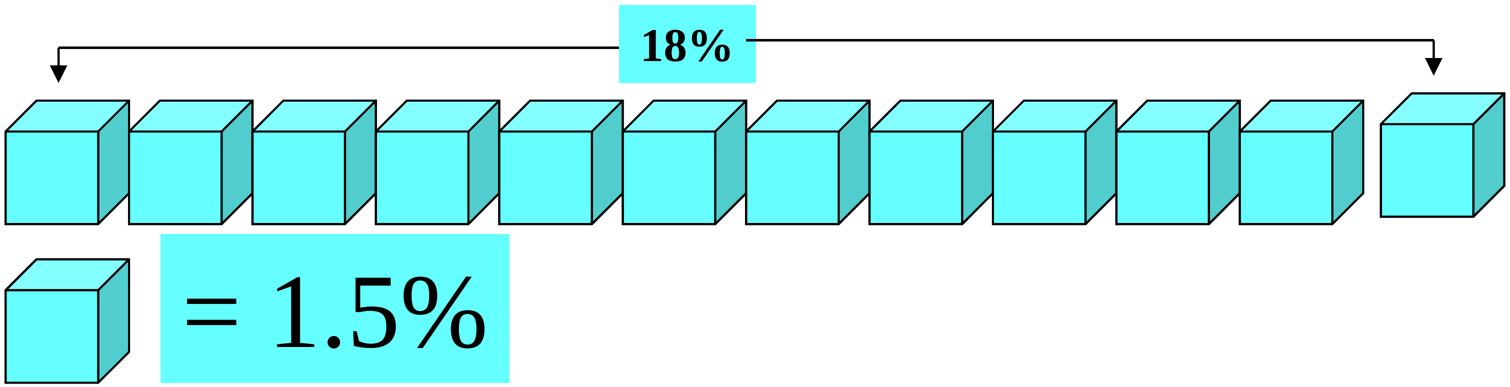
- **Question**: Suppose that you invest $1 for 1 year at 18% compounded monthly. How much interest would you earn?

- **Solution**:

$$F = \$1(1+i)^{12} = \$1(1+0.015)^{12}$$

$$= \$1.1956$$

$$i_a = 0.1956 \text{ or } 19.56\%$$



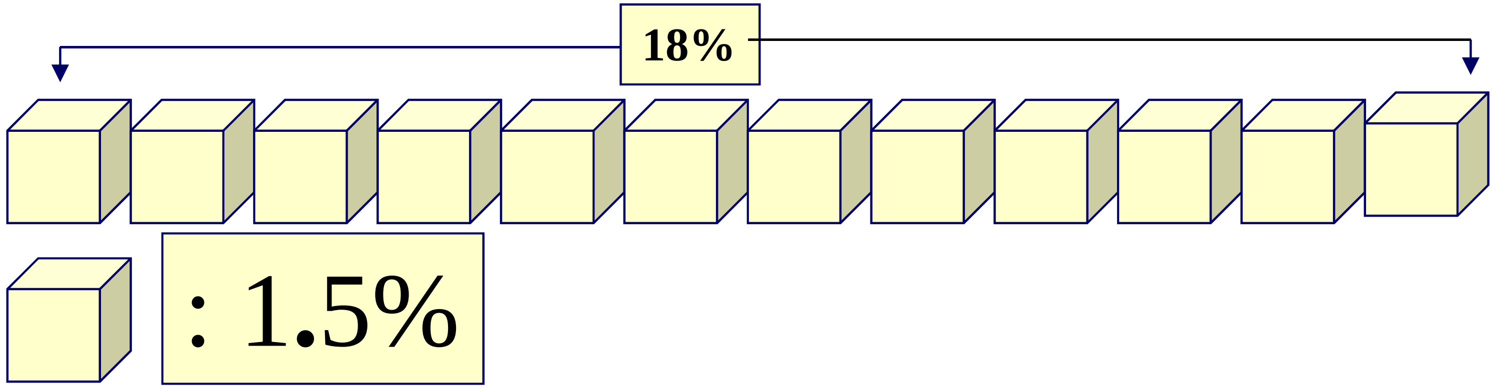
Effective Annual Interest Rate (Yield)

$$i_a = (1 + r/M)^M - 1$$

r = nominal interest rate per year

i_a = effective annual interest rate

M = number of interest periods per year

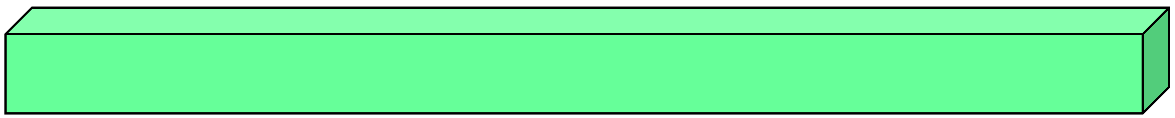


18% compounded **monthly**

or

1.5% per month for **12 months**

=



19.56 % compounded **annually**

Practice Problem (1)

- If your credit card calculates the interest based on 12.5% APR, what is your monthly interest rate and annual effective interest rate, respectively?

- Your current outstanding balance is $2,000 and skips payments for 2 months. What would be the total balance 2 months from now?

Solution

Monthly Interest Rate:

$$i = \frac{12.5\%}{12} = 1.0417\%$$

Annual Effective Interest Rate:

$$i_a = (1 + 0.010417)^{12} = 13.24\%$$

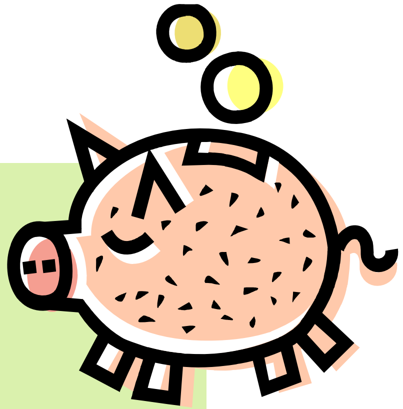
Total Outstanding Balance:

$$F = B_2 = \$2,000(F/P, 1.0417\%, 2)$$

$$= \$2,041.88$$

Practice Problem (2)

- Suppose your savings account pays 9% interest compounded quarterly. If you deposit $10,000 for one year, how much would you have?



(a) Interest rate per quarter:

$$i = \frac{9\%}{4} = 2.25\%$$

(b) Annual effective interest rate:

$$i_a = (1 + 0.0225)^4 - 1 = 9.31\%$$

(c) Balance at the end of one year (after 4 quarters)

$$\begin{aligned} F &= \$10,000(F/P, 2.25\%, 4) \\ &= \$10,000(F/P, 9.31\%, 1) \\ &= \$10,931 \end{aligned}$$

Effective Annual Interest Rates (9% compounded quarterly)

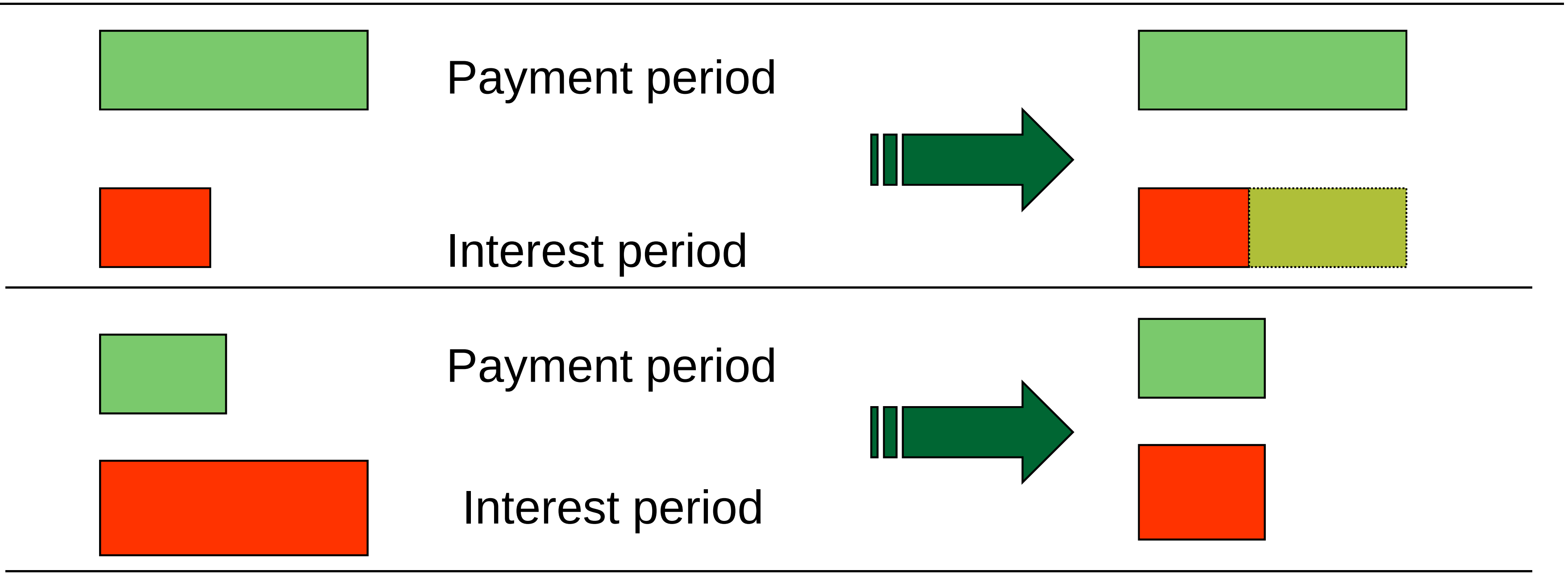
First quarter	**Base amount** **+ Interest (2.25%)**	**$10,000** **+ $225**
Second quarter	**= New base amount** **+ Interest (2.25%)**	**= $10,225** **+$230.06**
Third quarter	**= New base amount** **+ Interest (2.25%)**	**= $10,455.06** **+$235.24**
Fourth quarter	**= New base amount** **+ Interest (2.25 %)** **= Value after one year**	**= $10,690.30** **+ $240.53** **= $10,930.83**

Nominal and Effective Interest Rates with Different Compounding Periods

Effective Rates					
Nominal Rate	Compounding Annually	Compounding Semi-annually	Compounding Quarterly	Compounding Monthly	Compounding Daily
4%	4.00%	4.04%	4.06%	4.07%	4.08%
5	5.00	5.06	5.09	5.12	5.13
6	6.00	6.09	6.14	6.17	6.18
7	7.00	7.12	7.19	7.23	7.25
8	8.00	8.16	8.24	8.30	8.33
9	9.00	9.20	9.31	9.38	9.42
10	10.00	10.25	10.38	10.47	10.52
11	11.00	11.30	11.46	11.57	11.62
12	12.00	12.36	12.55	12.68	12.74

Nominal and Effective Interest Rates

- Why do we need an effective interest rate per payment period?
  - Whenever payment and compounding periods differ from each other, one or the other must be transformed so that both conform to the same unit of time



Effective Interest Rate per Payment Period (*i*)

$$i = [1 + r / CK]^{C} - 1$$

C = number of interest periods per payment period

K = number of payment periods per year

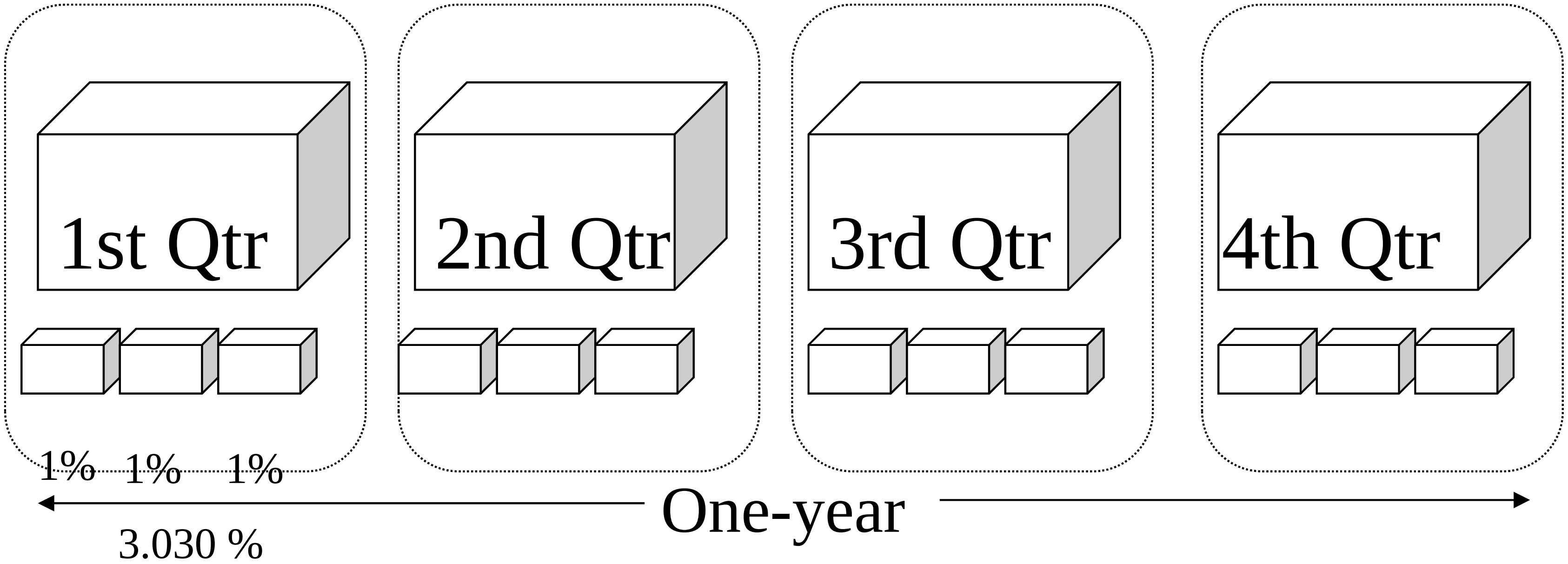
CK = total number of interest periods per year, or M

r / K = nominal interest rate per payment period

12% compounded monthly

Payment Period = Quarter

Compounding Period = Month



- **Effective interest rate per quarter**

$$i = (1 + 0.01)^3 - 1 = 3.030\ \%$$

- **Effective annual interest rate**

$$i_a = (1 + 0.01)^{12} - 1 = 12.68\%$$

$$i_a = (1 + 0.03030)^4 - 1 = 12.68\%$$

Effective Interest Rate per Payment Period with Continuous Compounding

$$i = [1 + r / CK]^C - 1$$

where CK = number of compounding periods per year

continuous compounding => $C \to \infty$

$$i = \lim[(1 + r / CK)^C - 1]$$
$$= (e^r)^{1/K} - 1$$

Example:
12% compounded continuously

(a) Effective interest rate per quarter

$$i = e^{0.12/4} - 1$$
$$= 3.045\% \text{ per quarter}$$

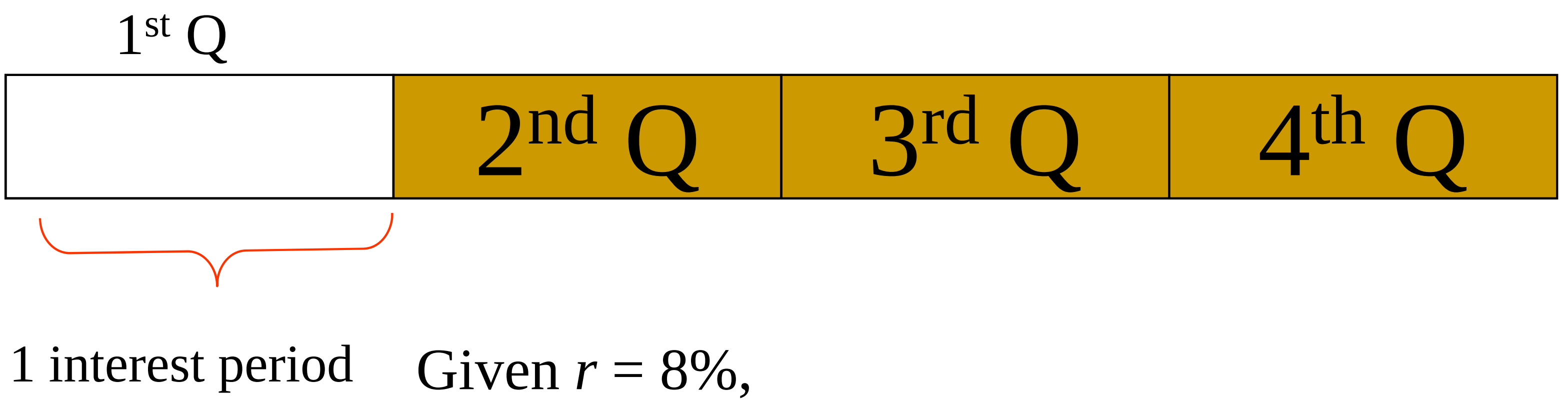
(b) Effective annual interest rate

$$i_a = e^{0.12/1} - 1$$
$$= 12.75\% \text{ per year}$$

Case 0: 8% compounded quarterly

Payment Period = Quarter

Interest Period = Quarterly



Given r = 8%,

K = 4 payments per year

C = 1 interest period per quarter

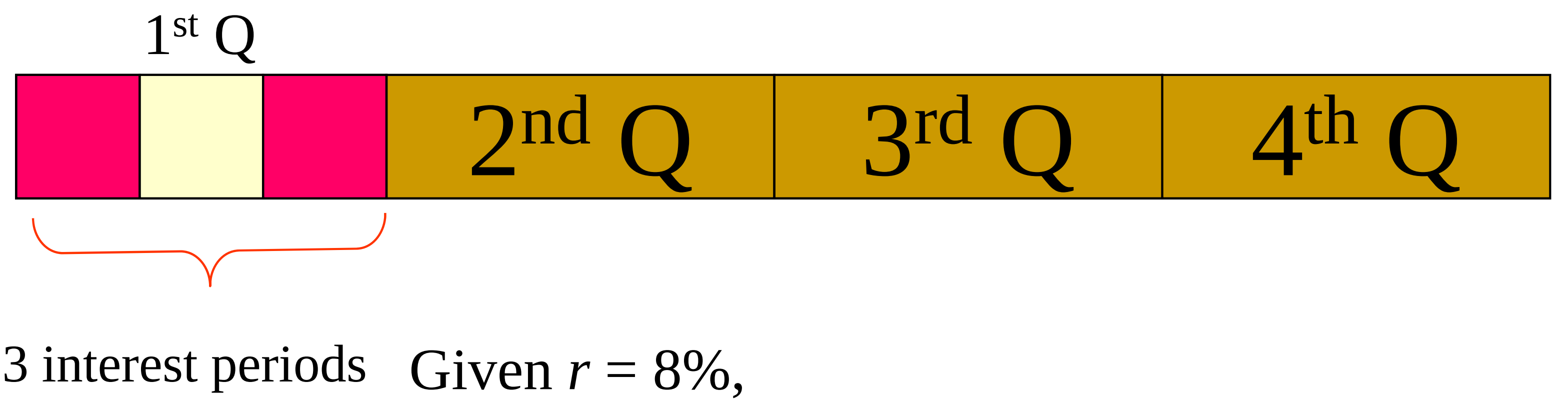
M = 4 interest periods per year

$$
\begin{aligned}
i &= [1 + r / CK]^{C} - 1 \\
&= [1 + 0.08 / (1)(4)]^{1} - 1 \\
&= 2.000\% \text{ per quarter}
\end{aligned}
$$

Case 1: 8% compounded monthly

Payment Period = Quarter

Interest Period = Monthly



Given r = 8%,

K = 4 payments per year

C = 3 interest periods per quarter

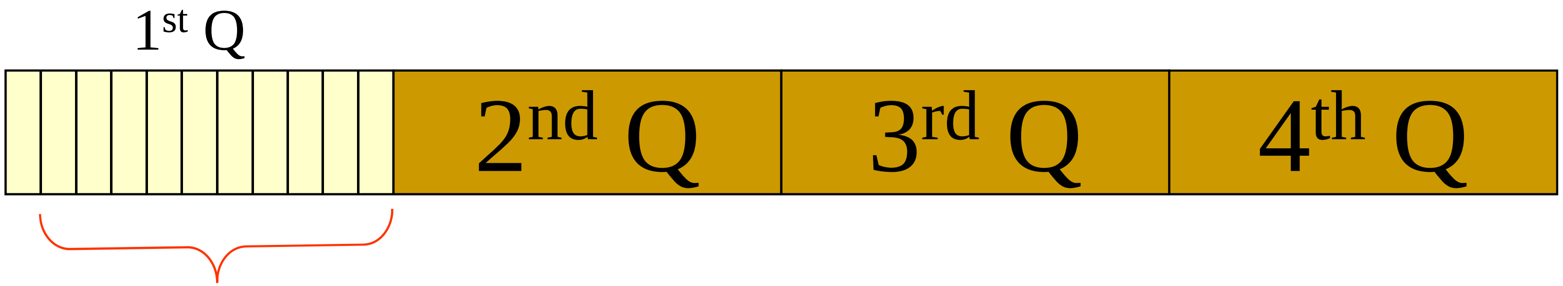
M = 12 interest periods per year

$$i = [1 + r / CK]^C - 1$$
$$= [1 + 0.08 / (3)(4)]^3 - 1$$
$$= 2.013\% \text{ per quarter}$$

Case 2: 8% compounded weekly

Payment Period = Quarter

Interest Period = Weekly



13 interest periods

Given r = 8%,

K = 4 payments per year

C = 13 interest periods per quarter

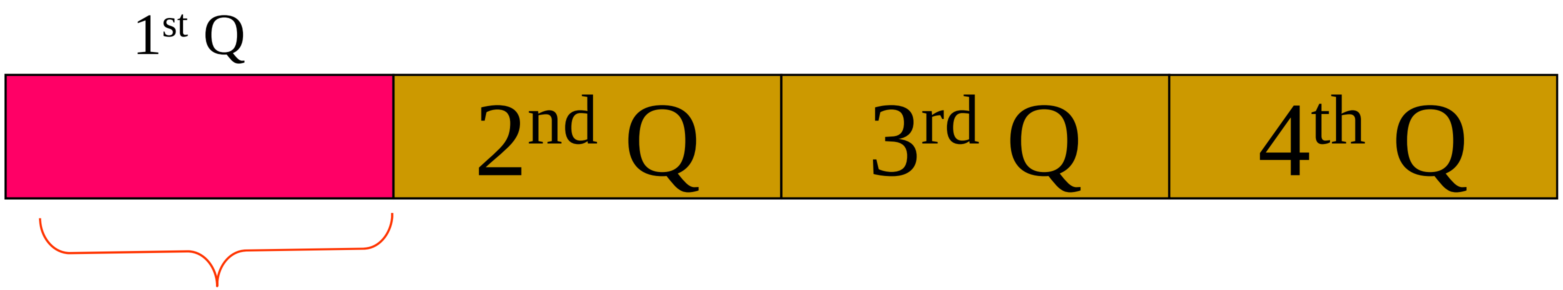
M = 52 interest periods per year

$$i = [1 + r / CK]^{C} - 1$$
$$= [1 + 0.08 / (13)(4)]^{13} - 1$$
$$= 2.0186\% \text{ per quarter}$$

Case 3: 8% compounded continuously

Payment Period = Quarter

Interest Period = Continuously



∞ interest periods

Given r = 8%,

K = 4 payments per year

$$i = e^{r/K} - 1$$
$$= e^{0.02} - 1$$
$$= 2.0201\ \%\ \text{per quarter}$$

Summary: Effective interest rate per quarter at Varying Compounding Frequencies

Case 0	Case 1	Case 2	Case 3
8% compounded quarterly	8% compounded monthly	8% compounded weekly	8% compounded continuously
Payments occur quarterly	Payments occur quarterly	Payments occur quarterly	Payments occur quarterly
2.000% per quarter	2.013% per quarter	2.0186% per quarter	2.0201% per quarter

Equivalence Calculations using Effective Interest Rates

- Step 1: Identify the payment period (e.g., annual, quarter, month, week, etc)

- Step 2: Identify the interest period (e.g., annually, quarterly, monthly, etc)

- Step 3: Find the effective interest rate that covers the **payment period.**

Case I: When Payment Period is Equal to Compounding Period

- Step 1: Identify the number of compounding periods (M) per year
- Step 2: Compute the effective interest rate per payment period (i)

$$i = r / M$$

- Step 3: Determine the total number of payment periods (N)

$$N = M \text{ (number of years)}$$

- Step 4: Use the appropriate interest formula using i and N above

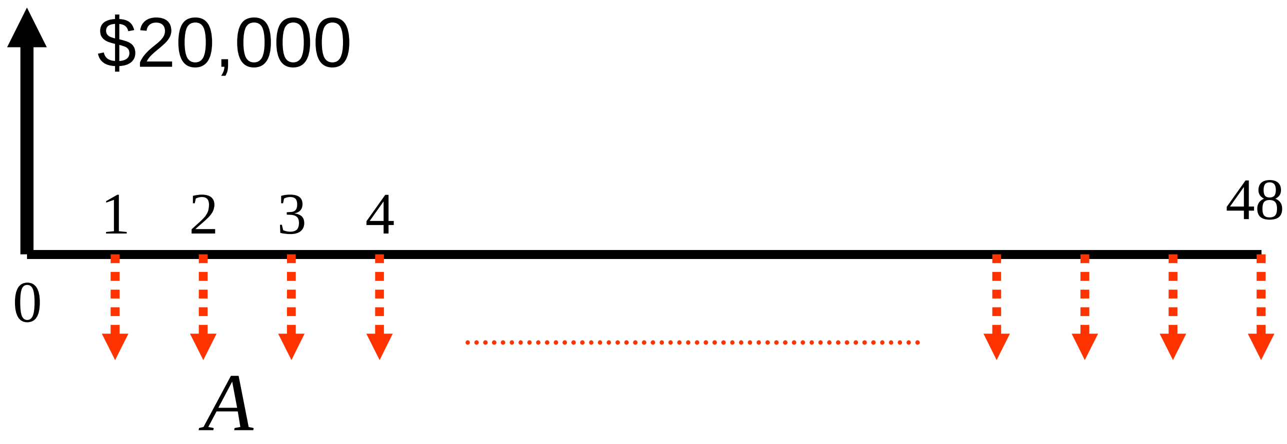
Example: Calculating Auto Loan Payments

Given:

- ❑Invoice Price = $21,599
- ❑Sales tax at 4% = $21,599 (0.04) = $863.96
- ❑Dealer's freight = $21,599 (0.01) = $215.99
- ❑Total purchase price = $22,678.95
- ❑Down payment = $2,678.95
- ❑Loan payment = $22,678.95 - $2,678.95 = $20K
- ❑Dealer's interest rate = 8.5% APR
- ❑Length of financing = 48 months

❑Find: the monthly payment (*A*)

Solution: Payment Period = Interest Period

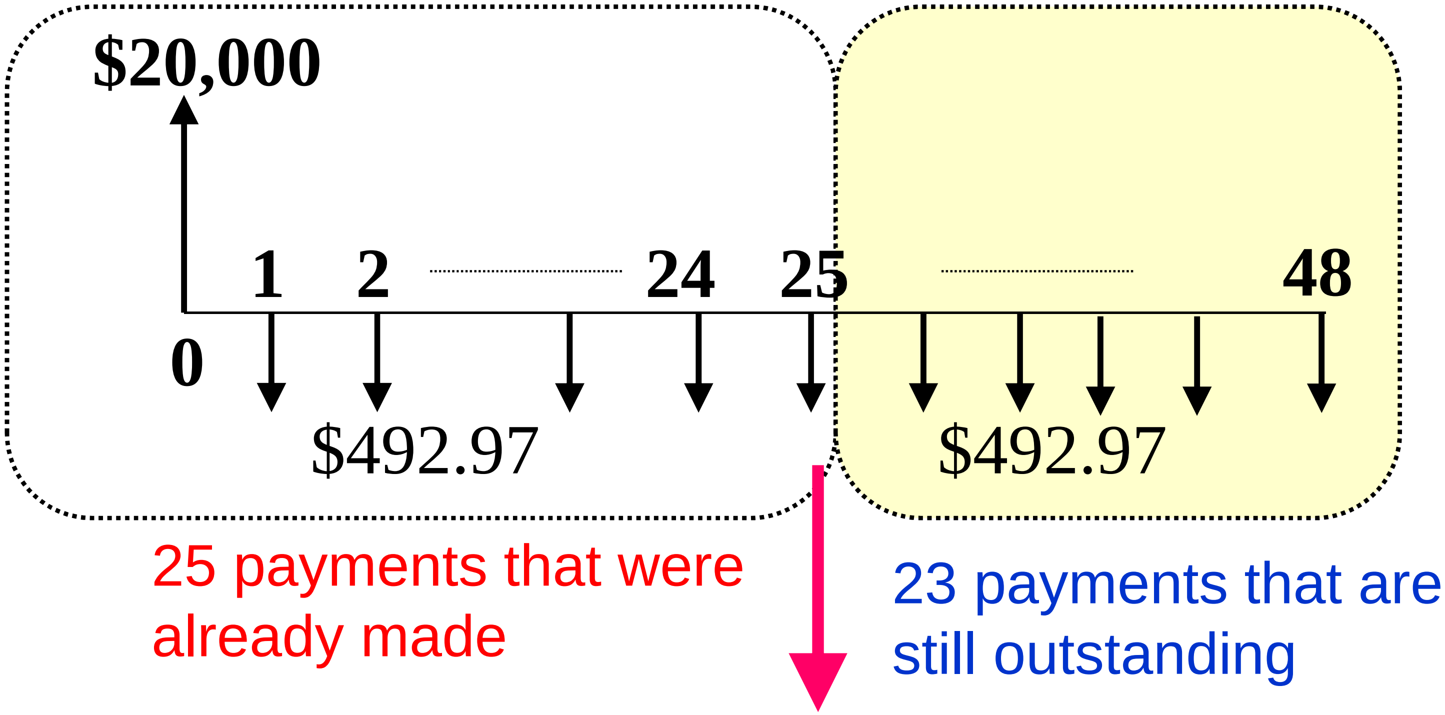


Given: P = \$20,000, r = 8.5% per year
K = 12 payments per year
N = 48 payment periods

Find A

- Step 1: $M = 12$
- Step 2: $i = r / M$ = 8.5% / 12 = 0.7083% per month
- Step 3: $N = (12)(4) = 48$ months
- Step 4: A = \$20,000(A/P, 0.7083%, 48) = \$492.97

Suppose you want to pay off the remaining loan in lump sum right after making the 25th payment. How much would this lump be?



$$P = \$492.97\ (P/A,\ 0.7083\%,\ 23)$$
$$= \$10,428.96$$

Practice Problem

- You have a habit of drinking a cup of Starbuck coffee (US$2.00 a cup) on the way to work every morning for 30 years. If you put the money in the bank for the same period, how much would you have, assuming your accounts earns 5% interest compounded daily.

- NOTE: Assume you drink a cup of coffee every day including weekends.

Solution

- Payment period: Daily
- Compounding period: Daily

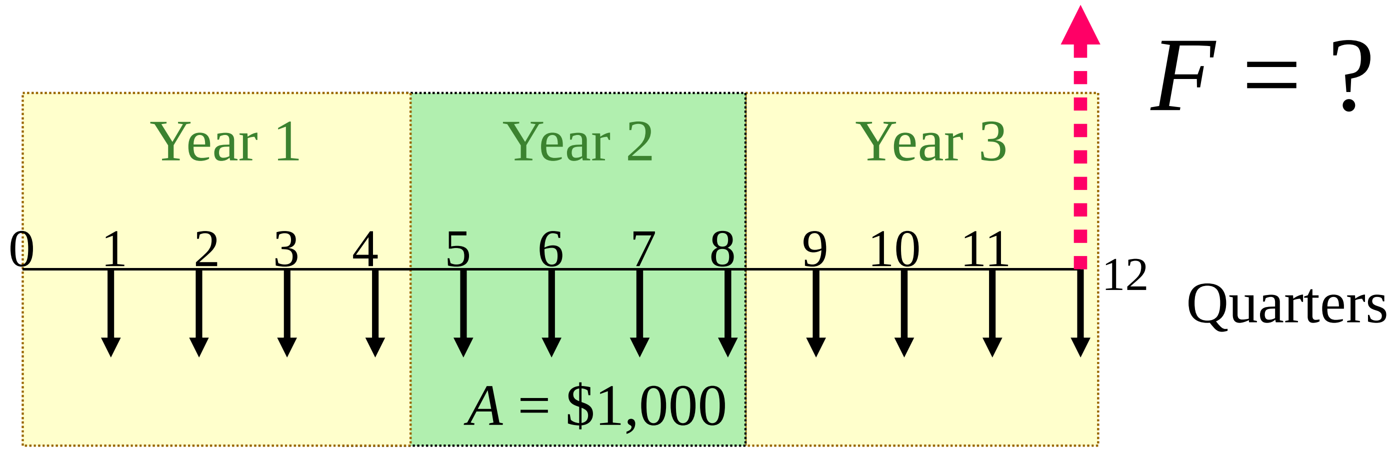
$$
\begin{aligned}
i &= \frac{5\%}{365} = 0.0137\% \text{ per day} \\
N &= 30 \times 365 = 10{,}950 \text{ days} \\
F &= \$2(F/A, 0.0137\%, 10950) \\
&= \$50{,}831
\end{aligned}
$$

Case II: When Payment Periods Differ from Compounding Periods

- Step 1: Identify the following parameters
  - M = No. of compounding periods
  - K = No. of payment periods
  - C = No. of interest periods per payment period
- Step 2: Compute the effective interest rate per payment period
  - For discrete compounding $i = [1 + r / CK]^C - 1$
  - For continuous compounding $i = e^{r/K} - 1$
- Step 3: Find the total no. of payment periods
  - $N = K$ (no. of years)
- Step 4: Use i and N in the appropriate equivalence formula

Example (1): Discrete Case: Quarterly deposits with Monthly compounding

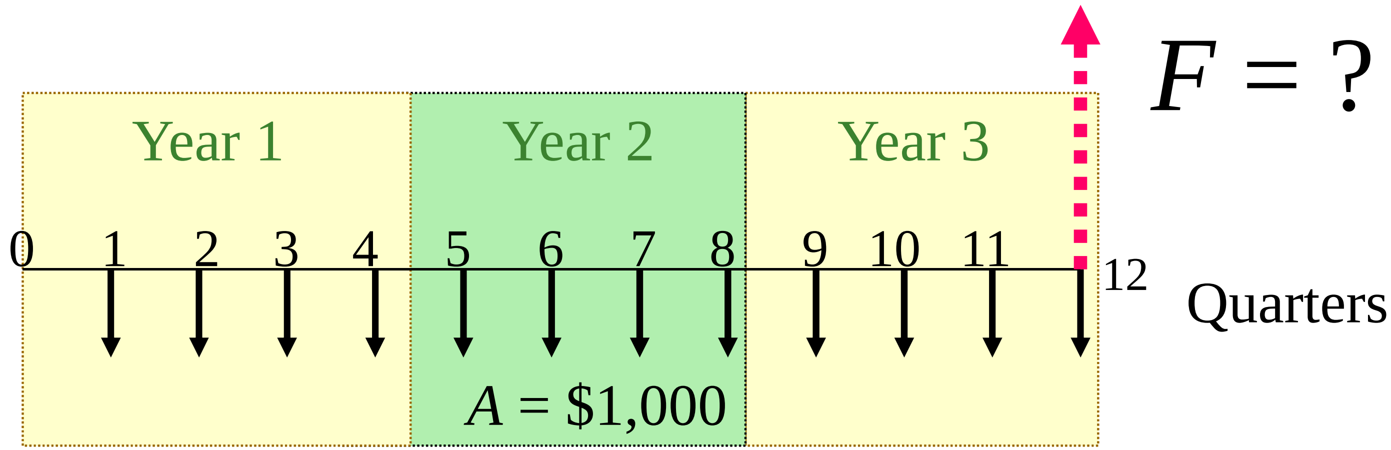
Given: A = \$1,000 per quarter, r = 12% per year, M = 12 compounding periods per year, and N = 3 years



- Step 1: M = 12 compounding periods/year
  K = 4 payment periods/year
  C = 3 interest periods per quarter
- Step 2: $i = [1 + 0.12/(3)(4)]^3 - 1$
  $= 3.030\%$
- Step 3: N = 4(3) = 12
- Step 4: F = \$1,000 (F/A, 3.030%, 12)
  = \$14,216.24

Example (2): Continuous Case: Quarterly deposits with Continuous compounding

Given: A = \$1,000 per quarter, r = 12% per year, M = 12 compounding periods per year, and N = 3 years



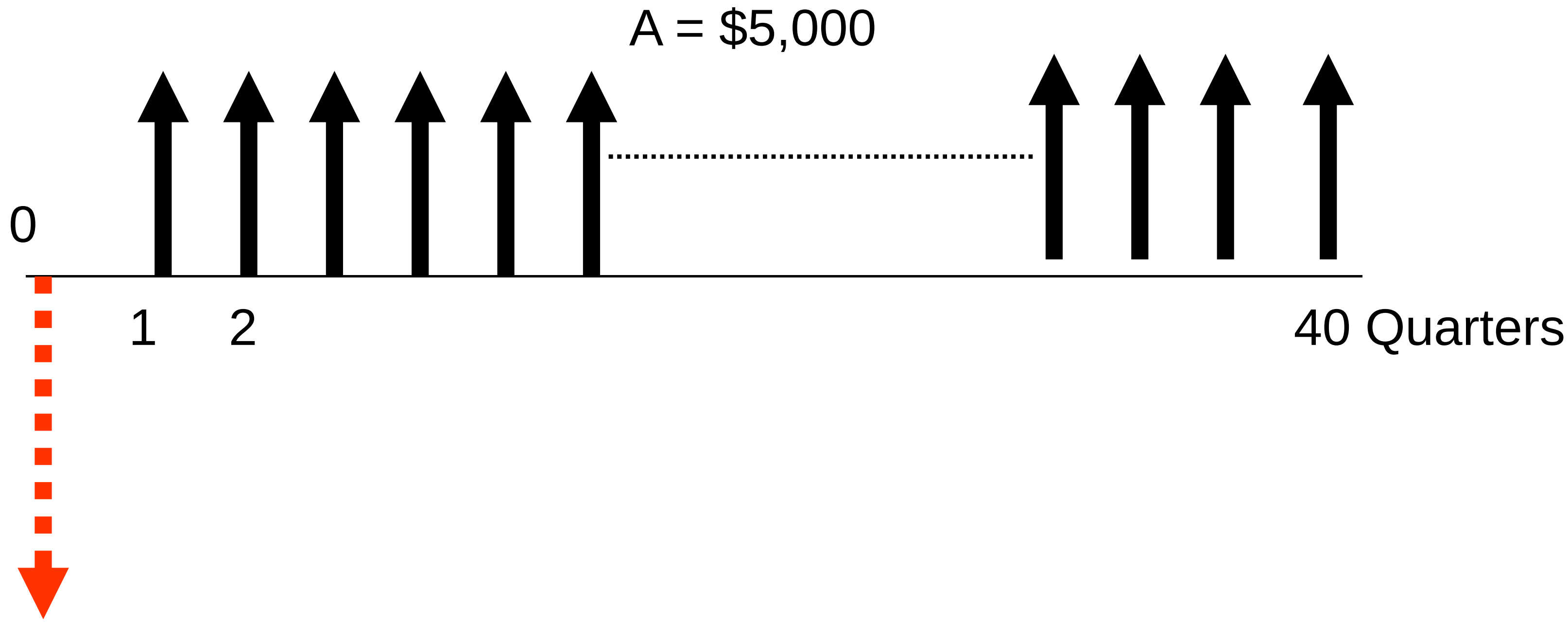
- Step 1: K = 4 payment periods/year
  $C = \infty$ interest periods per quarter
- Step 2: $i = e^{0.12/4} - 1$

  $= 3.045\%$ per quarter
- Step 3: N = 4(3) = 12
- Step 4: F = \$1,000 (F/A, 3.045%, 12)
  = \$14,228.37

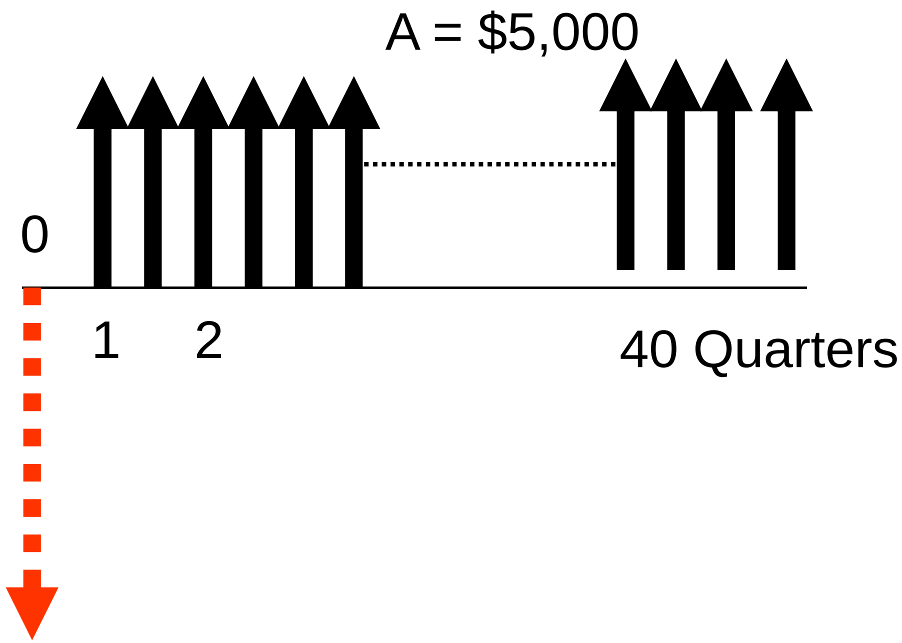
Practice Problem

- A series of equal quarterly payments of $5,000 for 10 years is equivalent to what present amount at an interest rate of 9% compounded
  - (a) quarterly
  - (b) monthly
  - (c) continuously

Solution



(a) Quarterly



- Payment period : Quarterly
- Interest Period: Quarterly

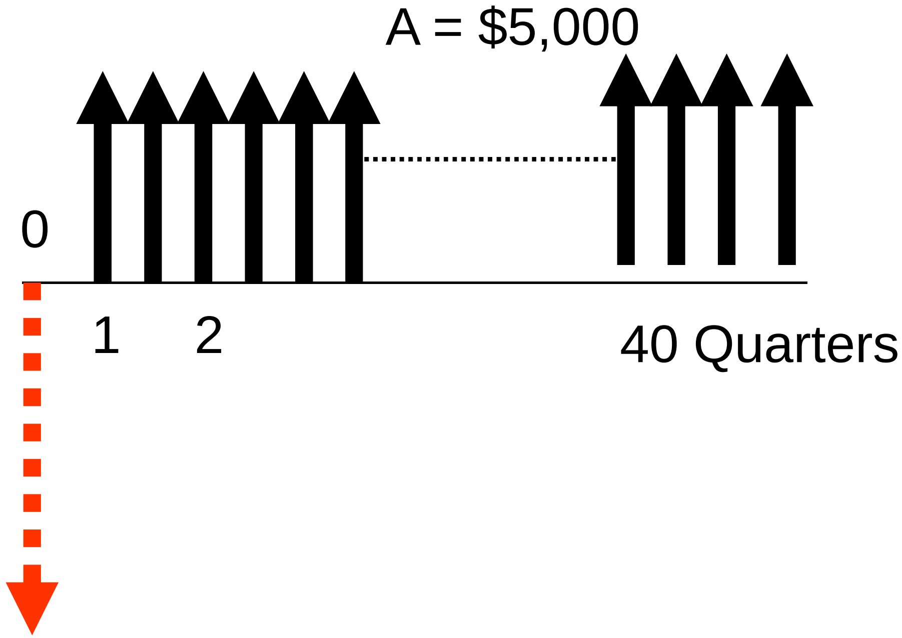
$$i = \frac{9\%}{4} = 2.25\% \text{ per quarter}$$

$$N = 40 \text{ quarters}$$

$$P = \$5{,}000(P/A, 2.25\%, 40)$$

$$= \$130{,}968$$

(b) Monthly



- Payment period : Quarterly
- Interest Period: Monthly

$$i = \frac{9\%}{12} = 0.75\% \text{ per month}$$

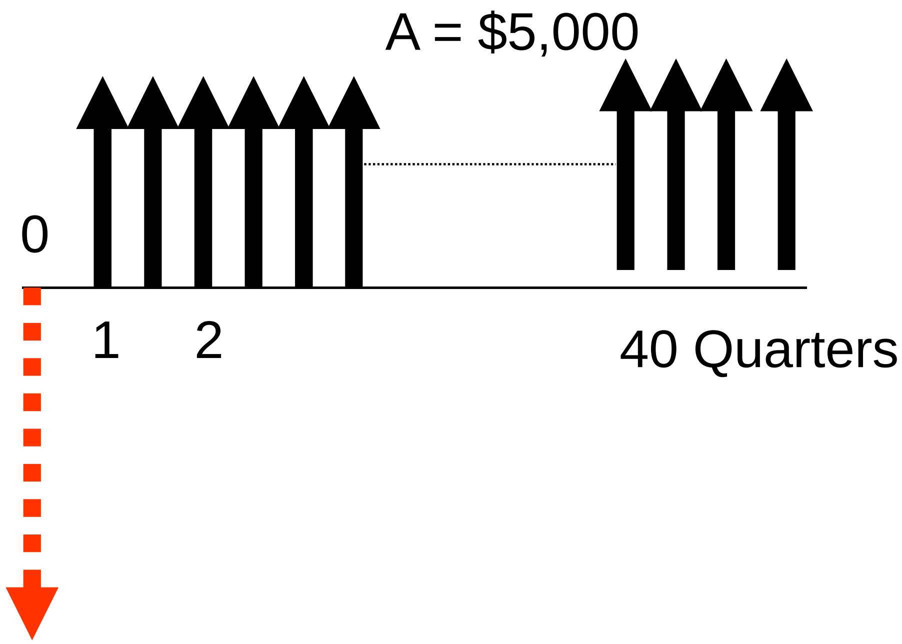
$$i_p = (1 + 0.0075)^3 = 2.267\% \text{ per quarter}$$

$$N = 40 \text{ quarters}$$

$$P = \$5,000(P/A, 2.267\%, 40)$$

$$= \$130,586$$

(c) Continuously



- Payment period : Quarterly
- Interest Period: Continuously

$$i = e^{0.09/4} - 1 = 2.276\% \text{ per quarter}$$

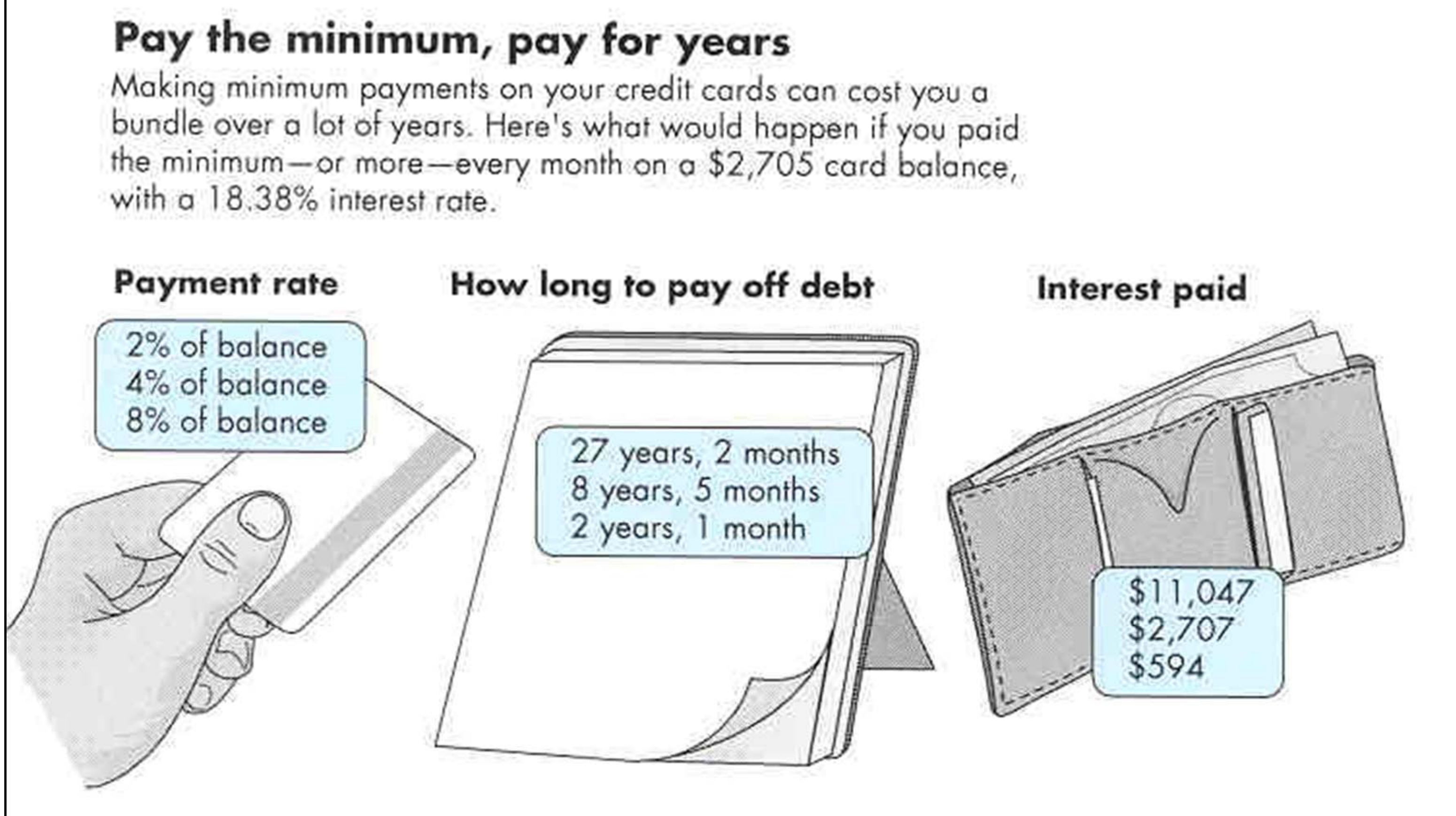
$$N = 40 \text{ quarters}$$

$$P = \$5,000(P/A, 2.276\%, 40)$$

$$= \$130,384$$

Debt Management

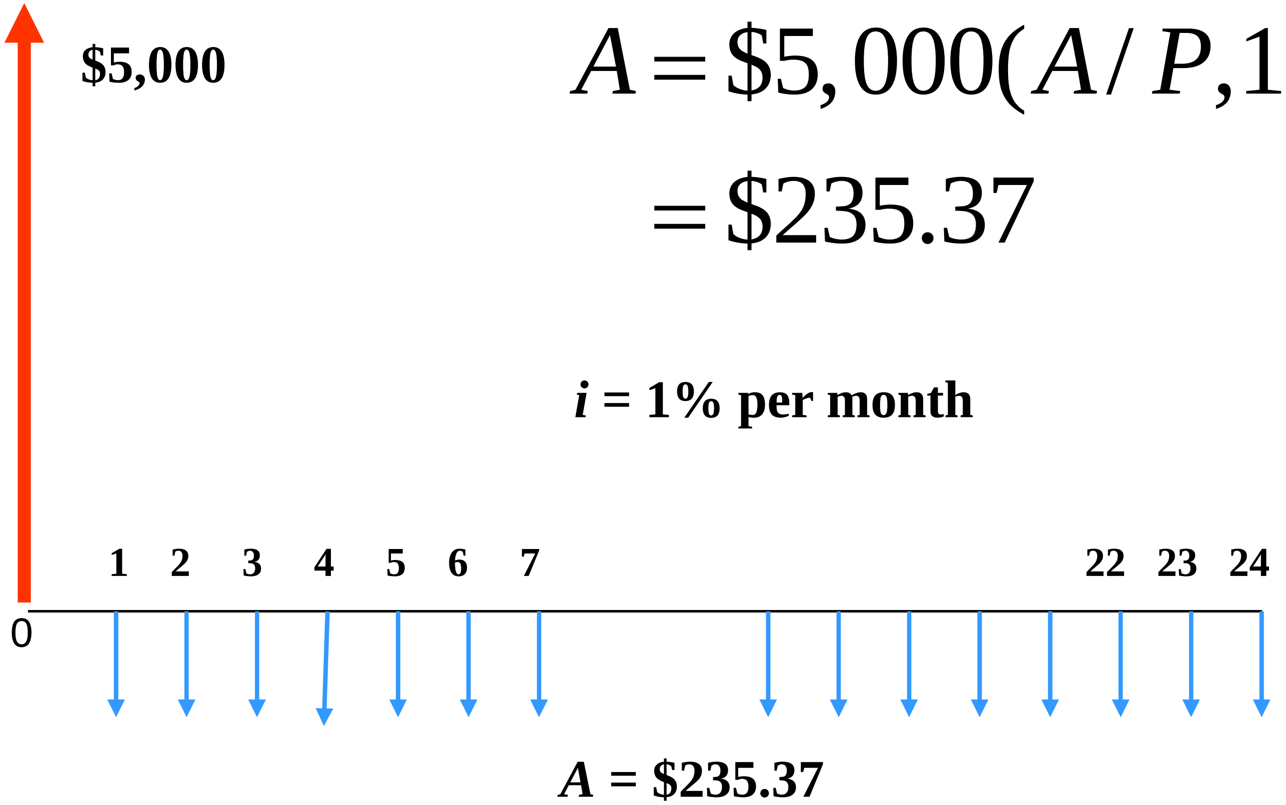
Credit card debt and commercial loans are among the most significant financial transactions involving interest.



(Source: *USA Today*, April 21, 1998, © *USA Today*, used with permission)

Example (1): Loan Repayment Schedule

$$A = \$5,000(A/P, 1\%, 24)$$
$$= \$235.37$$

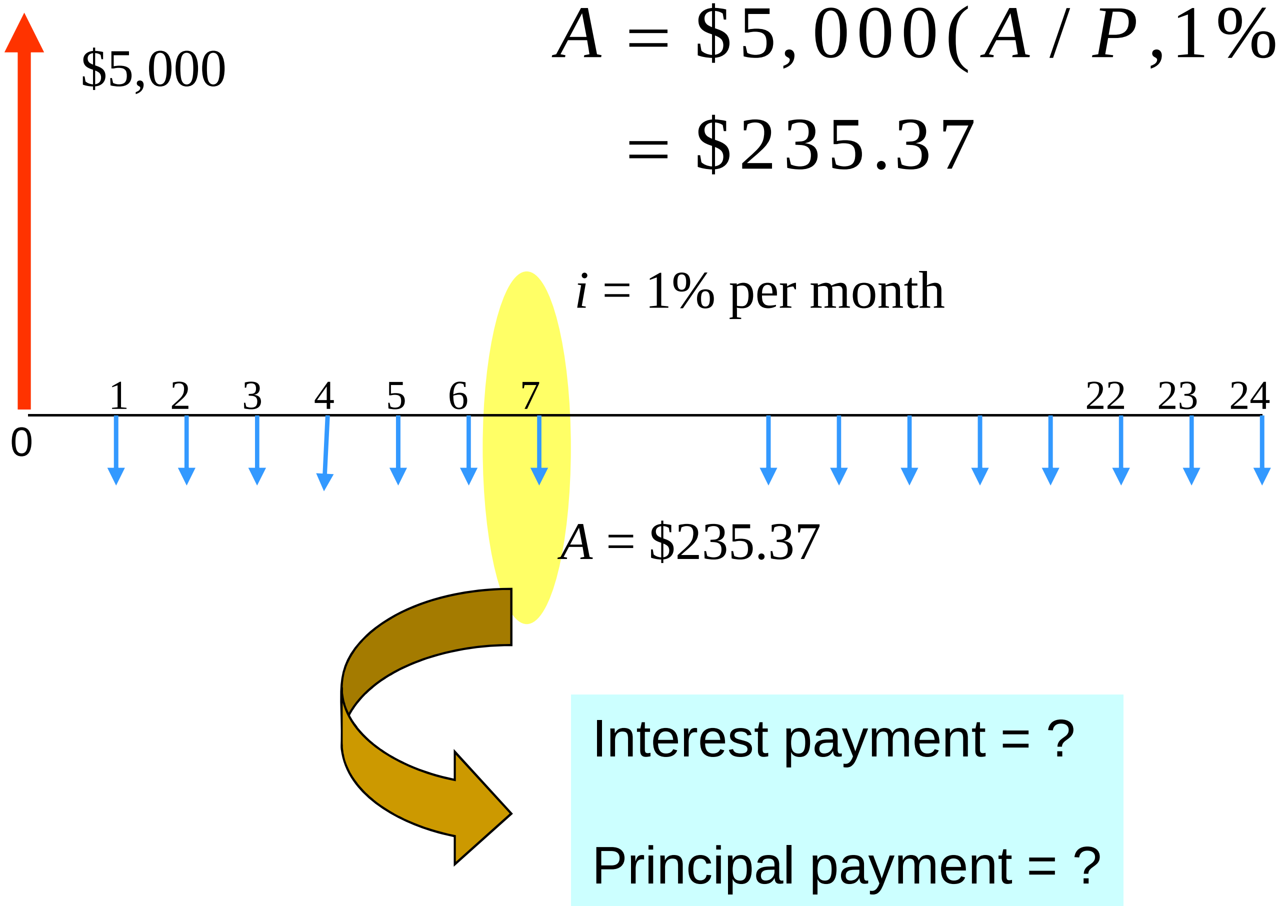


Practice Problem

- Consider the 7th payment ($235.37)
- (a) How much is the interest payment?
- (b) What is the amount of principal payment?

Solution

$$A = \$5,000(A/P, 1\%, 24)$$
$$= \$235.37$$



Solution

- Outstanding balance at the end of period 6: (Note: 18 outstanding payments)

$$B_6 = \$235.37(P/A, 1\%, 18) = \$3,859.66$$

- Interest payment for period 7:

$$IP_7 = \$3,859.66(0.01) = \$38.60$$

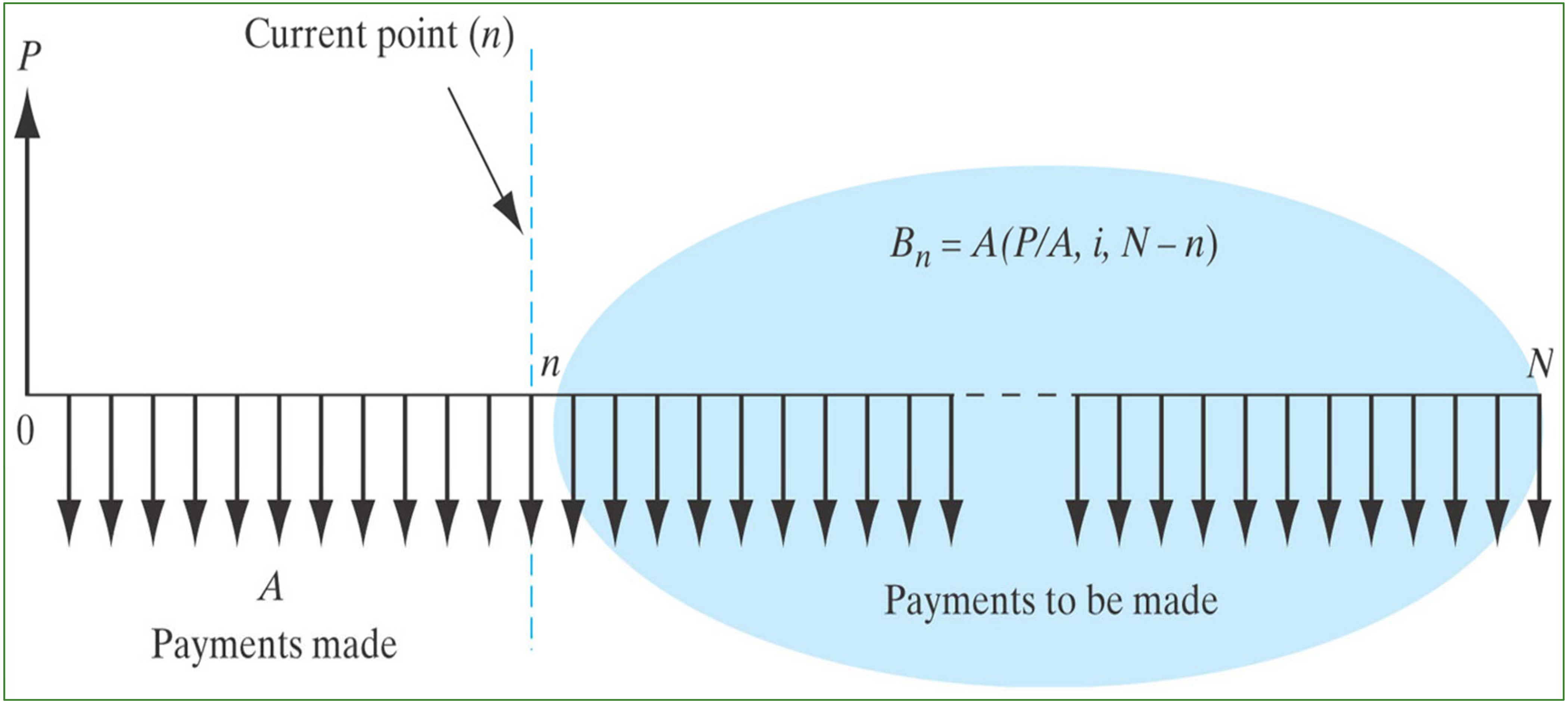
- Principal payment for period 7:

$$PP_7 = \$235.37 - \$38.60 = \$196.77$$

Note: $IP_7 + PP_7 = \$235.37$

	A	B	C	D	E	F	G
1							
2							
3	Example 3.7 Loan Repayment Schedule						
4							
5	Contract amount	$ 5,000.00		Total payment		$ 5,648.82	
6	Contract period	24		Total interest		$648.82	
7	APR (%)	12					
8	Monthly Payment	($235.37)					
9							
10		Payment No.	Payment Size	Principal Payment	Interest payment	Loan Balance	
11		1	($235.37)	($185.37)	($50.00)	$4,814.63	
12		2	($235.37)	($187.22)	($48.15)	$4,627.41	
13		3	($235.37)	($189.09)	($46.27)	$4,438.32	
14		4	($235.37)	($190.98)	($44.38)	$4,247.33	
15		5	($235.37)	($192.89)	($42.47)	$4,054.44	
16		6	($235.37)	($194.82)	($40.54)	$3,859.62	
17		7	($235.37)	($196.77)	($38.60)	$3,662.85	
18		8	($235.37)	($198.74)	($36.63)	$3,464.11	
19		9	($235.37)	($200.73)	($34.64)	$3,263.38	
20		10	($235.37)	($202.73)	($32.63)	$3,060.65	
21		11	($235.37)	($204.76)	($30.61)	$2,855.89	
22		12	($235.37)	($206.81)	($28.56)	$2,649.08	
23		13	($235.37)	($208.88)	($26.49)	$2,440.20	
24		14	($235.37)	($210.97)	($24.40)	$2,229.24	
25		15	($235.37)	($213.08)	($22.29)	$2,016.16	
26		16	($235.37)	($215.21)	($20.16)	$1,800.96	
27		17	($235.37)	($217.36)	($18.01)	$1,583.60	
28		18	($235.37)	($219.53)	($15.84)	$1,364.07	
29		19	($235.37)	($221.73)	($13.64)	$1,142.34	
30		20	($235.37)	($223.94)	($11.42)	$918.40	
31		21	($235.37)	($226.18)	($9.18)	$692.21	
32		22	($235.37)	($228.45)	($6.92)	$463.77	
33		23	($235.37)	($230.73)	($4.64)	$233.04	
34		24	($235.37)	($233.04)	($2.33)	$0.00	
35							

Calculating the Remaining Loan Balance after Making the *n*th Payment

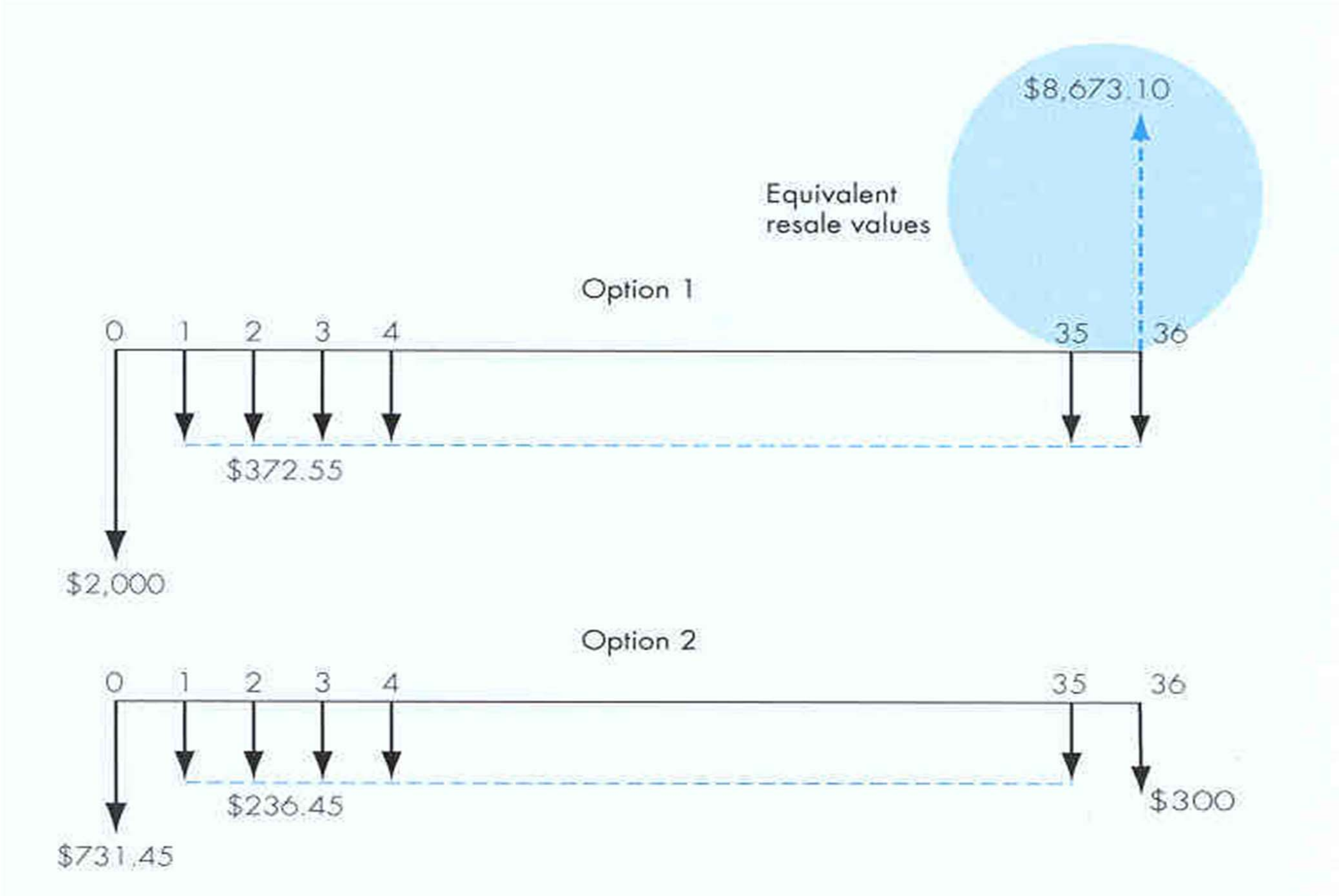


The interest payment in period n is, $I_n = i \times B_{n-1} = A \times (P/A, i, N-n+1) \times i$

Example (2): Buying versus Lease Decision

	Option 1 Debt Financing	Option 2 Lease Financing
Price	$14,695	$14,695
Down payment	$2,000	0
APR (%)	3.6%	
Monthly payment	$372.55	$236.45
Length	36 months	36 months
Fees		$495
Cash due at lease end		$300
Purchase option at lease end		$8.673.10
Cash due at signing	$2,000	$731.45

Which Interest Rate to Use to Compare These Options?



Your Earning Interest Rate = 6%

- Debt Financing:

  $$P_{debt} = \$2{,}000 + \$372.55(P/A, 0.5\%, 36) - \$8{,}673.10(P/F, 0.5\%, 36) = \$6{,}998.47$$

- Lease Financing:

  $$P_{lease} = \$495 + \$236.45 + \$236.45(P/A, 0.5\%, 35) + \$300(P/F, 0.5\%, 36) = \$8{,}556.90$$

Inflation and Economic Analysis

- What is inflation?
- How do we measure inflation?
- How do we incorporate the effect of inflation in equivalence calculation?

What is Inflation?

Inflation is the rate at which the general level of prices and goods and services is rising, and subsequently, purchasing power is falling.

❑Time Value of Money

- Earning Power How much you currently make at your place of employment plays a major part in your earning power
- Purchasing Power The value of a currency expressed in terms of the amount of goods or services that one unit of money can buy

❑Earning Power

- Investment opportunities

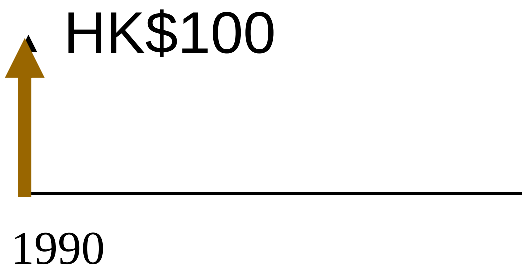
❑Purchasing Power

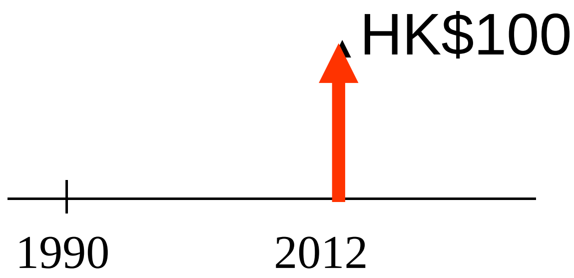
- Decrease in purchasing power (inflation) 通貨膨脹
- Increase in purchasing power (deflation) 通貨緊縮

Earning Power

- **True Earning Power** = (Monthly Income - Monthly Taxes and Necessity Expenses) / Time
- For example: John makes $15,000 a month. His taxes and living expenses total $12,000 a month. He usually wake up at 6:30 AM to get ready for work, and return home around 6:30 PM each day; totaling about 12 hours per day, 60 hours per week, or approximately 260 hours per month. Using the equation above, John's **true earning power is only $11.54 per hour!**

Inflation - Decrease in Purchasing Power





You could buy 11.6 Big Macs in year 1990.

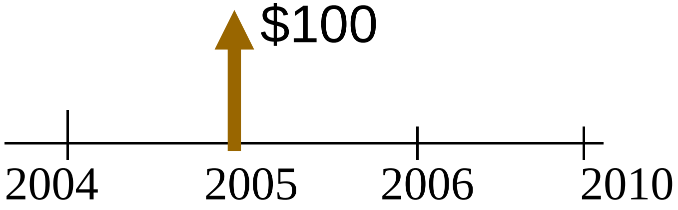
You can only buy 6.1 Big Macs in year 2012.

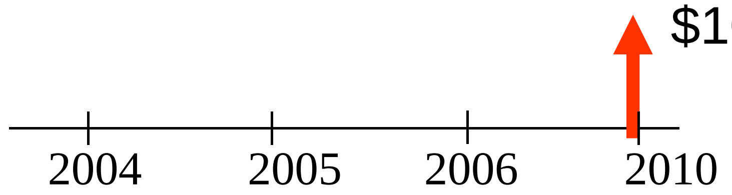
HK$8.60 / unit → +92% **Price change due to inflation** → HK$16.50 / unit

The $100 in year 2012 has only $52 worth purchasing power of 1990

(Data source: http://bigmacindex.org/2012-big-mac-index.html)

Deflation - Increase in Purchasing Power





You could purchase <u>63.69</u> gallons of purified drink water 5 years ago.

You can now purchase <u>80</u> gallons of purified drink water.

$1.57 / gallon → -20.38% → $1.25 / gallon

Price change due to deflation

Inflation Terminology - I

- Producer Price Index (PPI): a statistical measure of industrial price change, compiled monthly by the Statistics Bureau of the government department
- Consumer Price Index (CPI): a statistical measure of change, over time, of the prices of goods and services in major expenditure groups-such as food, housing, apparel, transportation, and medical care - typically purchased by city consumers
- Average Inflation Rate (f): a single rate that accounts for the effect of varying yearly inflation rates over a period of several years
- General Inflation Rate ($\bar{f}$): the average inflation rate calculated based on the CPI for all items in the market basket

Inflation Rate in Hong Kong

(Source: http://www.tradingeconomics.com/hong-kong/inflation-cpi)

Inflation Rate in Mainland China

(Source: http://www.tradingeconomics.com/china/inflation-cpi)

Measuring Inflation

Consumer Price Index (CPI): the CPI compares the cost of a sample "market basket" of goods and services in a specific period relative to the cost of the same "market basket" in an earlier reference period. This reference period is designated as the **base period.**

Market basket

Base Period (1982-84)		2010
$100	→	$216.7

CPI for 2010 = 216.7

Average Inflation Rate (*f*)

Fact: Base Price = \$100 (year 0)
Inflation rate (year 1) = 4%
Inflation rate (year 2) = 8%
Average inflation rate over 2 years?

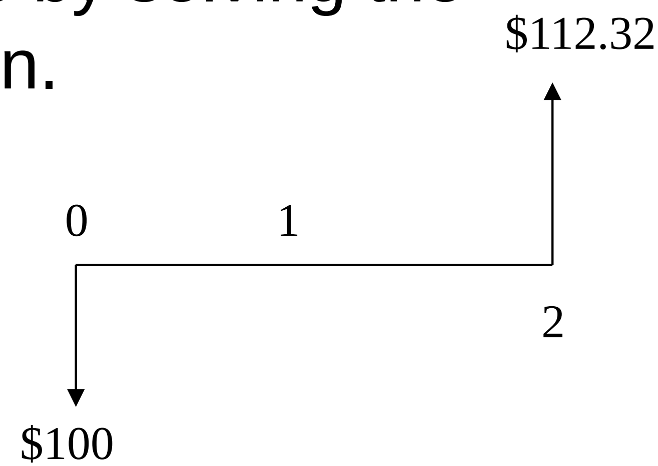
Step 1: Find the actual inflated price at the end of year 2.

$$\$100\,(1 + 0.04)\,(1 + 0.08) = \$112.32$$

Step 2: Find the average inflation rate by solving the following equivalence equation.

$$\$100\,(1 + f)^2 = \$112.32$$

$$f = 5.98\%$$



General Inflation Rate ($\bar{f}$)

Average inflation rate based on the CPI

$$CPI_n = CPI_0(1+\bar{f})^n,$$

$$\bar{f} = \left[\frac{CPI_n}{CPI_0}\right]^{1/n} - 1$$

where $\bar{f}$ = The genreal inflation rate,

CPI_n = The consumer price index at the end period n,

CPI_0 = The consumer price index for the base period.

Calculation:

Given: CPI for 2009 = 213.2,

CPI for 2000 = 172.2,

Find: $\bar{f}$

$$\bar{f} = \left[\frac{213.2}{172.2}\right]^{1/9} - 1$$

$$= 2.40\%$$

Example: Yearly and Average Inflation Rates

Year	Cost
0	$504,000
1	538,000
2	577,000
3	629,500

What are the annual inflation rates and the average inflation rate over 3 years?

Solution

Inflation rate during year 1 (f_1):

($538,400 - $504,000) / $504,000 = <u>6.83%.</u>

Inflation rate during year 2 (f_2):

($577,000 - $538,400) / $538,400 = <u>7.17 %.</u>

Inflation rate during year 3 (f_3):

($629,500 - $577,000) / $577,000 = <u>9.10%.</u>

The average inflation rate over 3 years is

$$f = \left(\frac{\$629{,}500}{\$504{,}000}\right)^{1/3} - 1 = 0.0769 = \boxed{7.69\%}$$

Inflation Terminology – II

The effect of inflation into economic analysis

- Actual Dollars (A_n):

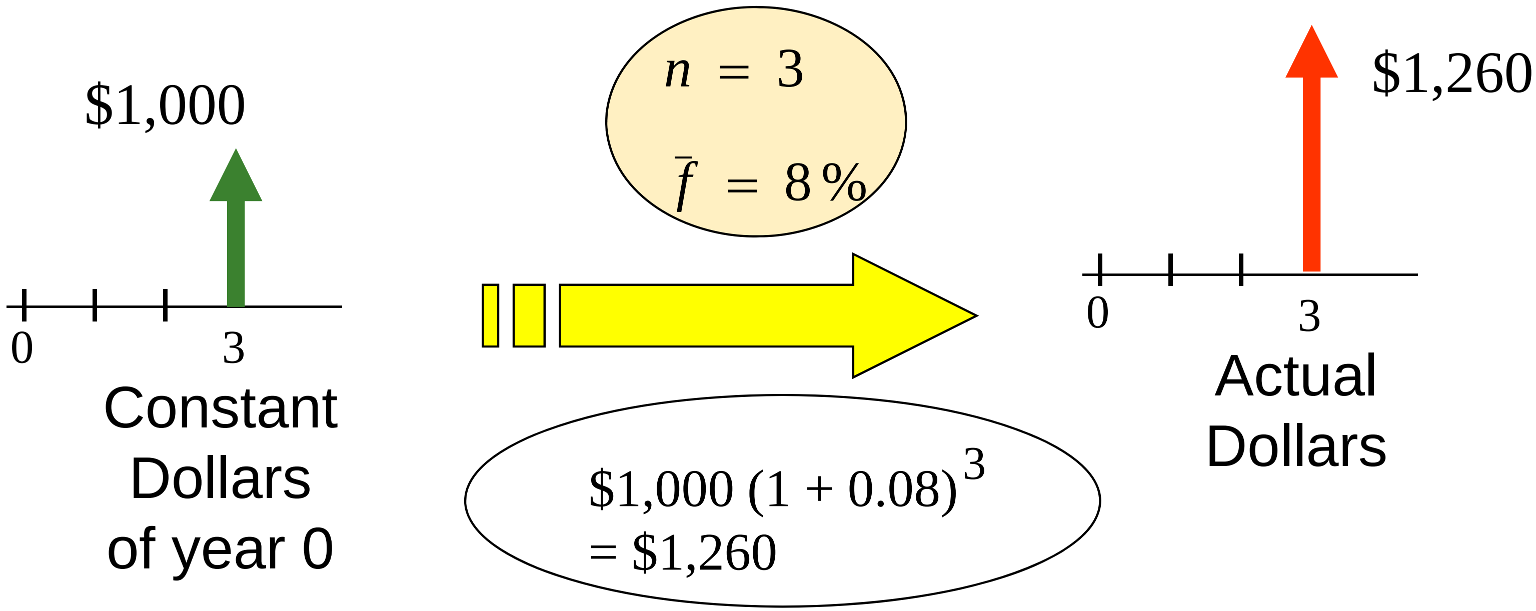
  Estimates of future cash flows for year ***n*** that take into account any anticipated changes in amount caused by inflationary or deflationary effects. Usually, these amounts are determined by applying an inflation rate to base-year dollar estimates.

- Constant (real) Dollars (A'_n):

  Represents constant purchasing power independent of the passage of time. We will assume that the base year is always time zero unless we specify otherwise.

Conversion from Constant to Actual Dollars

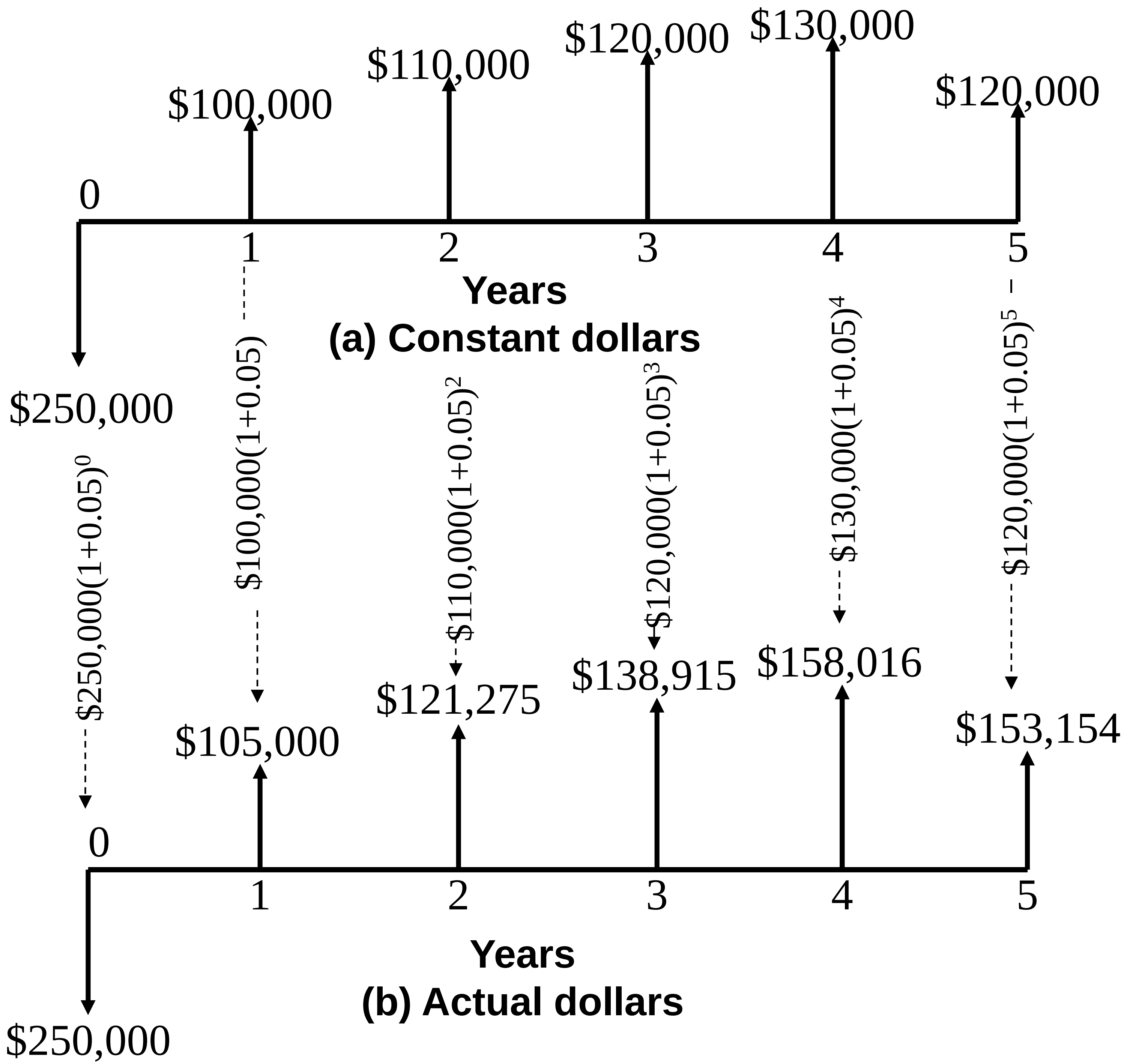
$$A_n = A'_n(1+\bar{f})^n \leftrightarrow A'_n(F/P,\bar{f},n)$$



Example: Conversion from Constant to Actual Dollars

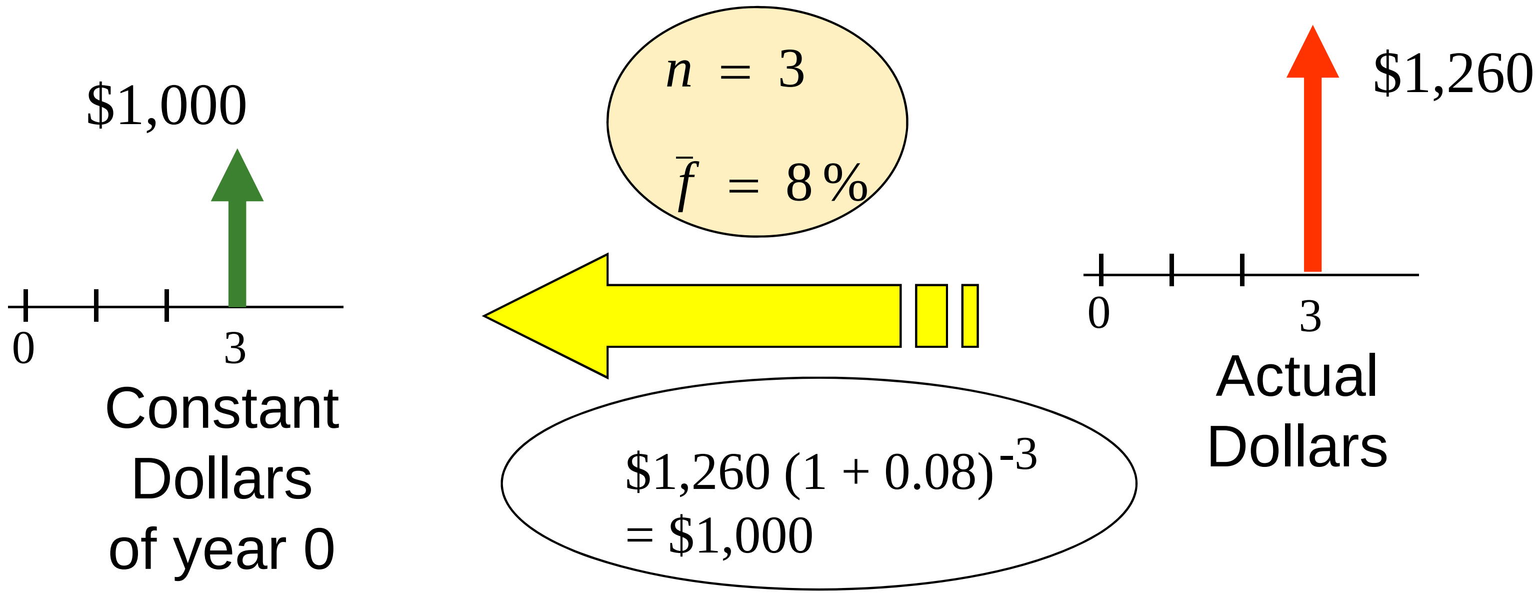
General inflation rate = 5%

Period	Net Cash Flow in Constant $	Conversion Factor	Cash Flow in Actual $
0	-$250,000	$(1+0.05)^0$	-$250,000
1	100,000	$(1+0.05)^1$	105,000
2	110,000	$(1+0.05)^2$	121,275
3	120,000	$(1+0.05)^3$	138,915
4	130,000	$(1+0.05)^4$	158,016
5	120,000	$(1+0.05)^5$	153,154


$100,000
$110,000
$120,000
$130,000
$120,000
0
1
2
3
4
5
Years
(a) Constant dollars
$250,000
$250,000(1+0.05)^0
$100,000(1+0.05)
$110,000(1+0.05)^2
$120,000(1+0.05)^3
$130,000(1+0.05)^4
$120,000(1+0.05)^5
$105,000
$121,275
$138,915
$158,016
$153,154
0
1
2
3
4
5
Years
(b) Actual dollars
$250,000

Conversion from Actual to Constant Dollars

$$A'_n = A_n(1+\bar{f})^{-n} \leftrightarrow A_n(P/F,\bar{f},n)$$



Example: Conversion from Actual to Constant Dollars

General inflation rate = 5%

End of period	Cash Flow in Actual $	Conversion at $\bar{f}$ = 5%	Cash Flow in Constant $	Loss in Purchasing Power
0	-$20,000	$(1+0.05)^{0}$	-$20,000	0%
1	20,000	$(1+0.05)^{-1}$	-19,048	4.76
2	20,000	$(1+0.05)^{-2}$	-18,141	9.30
3	20,000	$(1+0.05)^{-3}$	-17,277	13.62
4	20,000	$(1+0.05)^{-4}$	-16,454	17.73

Equivalence Calculations Under Inflation

Types of Interest Rate	
Market Interest Rate (*i*)	Inflation-free Interest Rate (*i*')

Types of Cash Flows	
Estimated in Constant Dollars	Estimated in Actual Dollars

Types of Analysis Method	
Constant-Dollar Analysis	Actual-Dollar Analysis

Inflation Terminology - III

- Inflation-free Interest Rate (i'): an estimate of the true earning power of money when the inflation effects have been removed (also known as real interest rate).

- Market interest rate (i): commonly known as the nominal interest rate, which takes into account the combined effects of the earning value of capital (earning power) and any anticipated changes in purchasing power (also known as inflation-adjusted interest rate).

Inflation and Cash Flow Analysis

Constant Dollar analysis **(inflation free interest rate *i'*)**

- Estimate all future cash flows in constant dollars.
- Use i' as an interest rate to find equivalent worth.

Actual Dollar Analysis **(market interest rate *i*)**

- Estimate all future cash flows in actual dollars.
- Use i as an interest rate to find equivalent worth.

Constant Dollar (A'_n) Analysis

When do we prefer Constant Dollar Analysis?

- In the absence of inflation, all economic analyses up to this point is, in fact, constant dollar analysis.
- Constant dollar analysis is common in the evaluation of many long-term public projects, because government do no pay income taxes.
- For private sector, income taxes are charged based on taxable income in actual dollars, so the actual dollar analysis is more common.

Actual Dollars (A_n) Analysis

- **Method 1: Deflation Method**

Step 1: Bring all cash flows to have common purchasing power.
Step 2: Consider the earning power.

- **Method 2: Adjusted-discount Method**

Combine Steps 1 and 2 into one step.

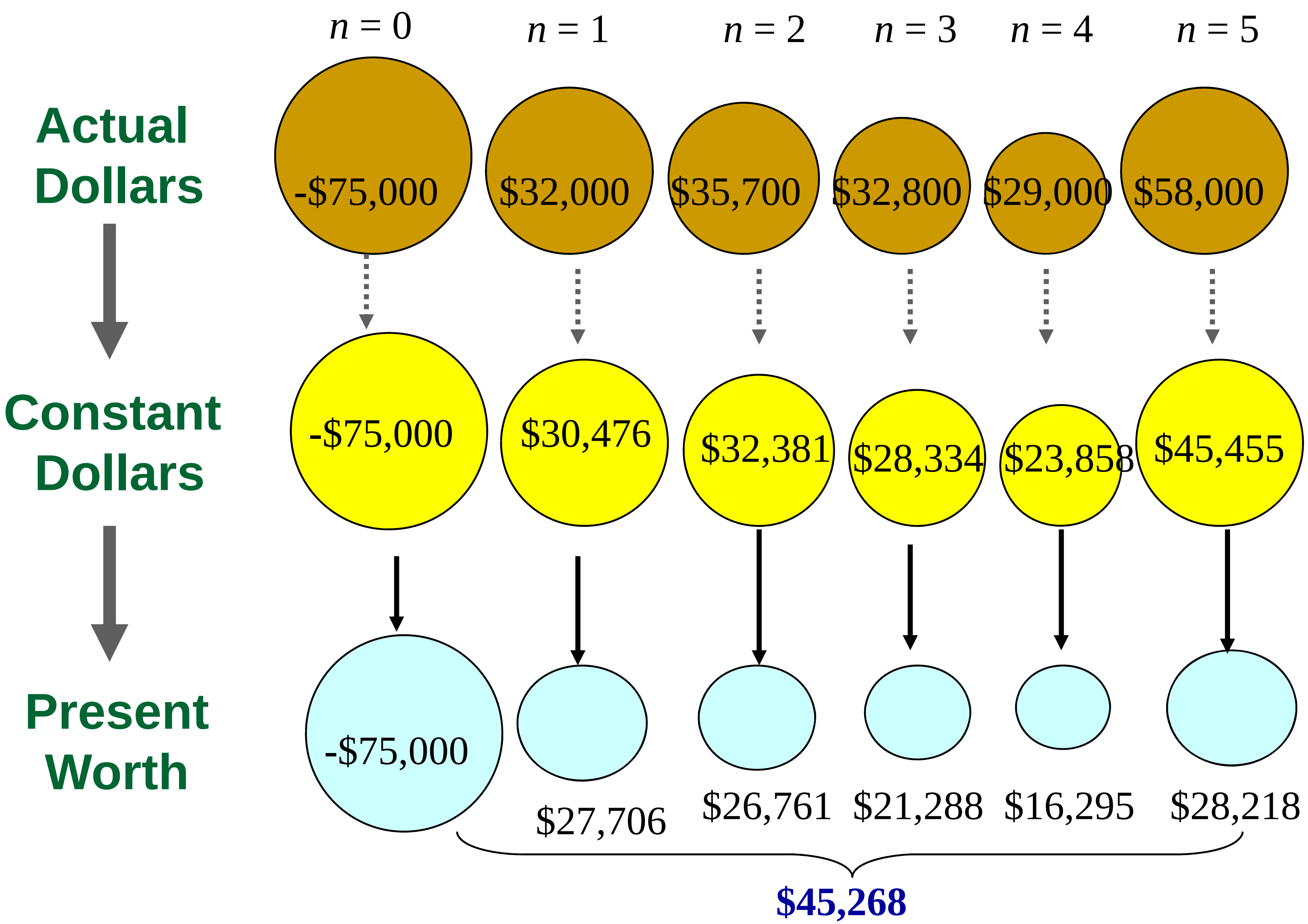
Example (1): Step 1: Convert actual dollars to Constant dollars

n	Cash Flows in Actual Dollars	Multiplied by Deflation Factor	Cash Flows in Constant Dollars
0	-$75,000	1	-$75,000
1	32,000	$(1+0.05)^{-1}$	30,476
2	35,700	$(1+0.05)^{-2}$	32,381
3	32,800	$(1+0.05)^{-3}$	28,334
4	29,000	$(1+0.05)^{-4}$	23,858
5	58,000	$(1+0.05)^{-5}$	45,445

Example (1): Step 2: Convert Constant dollars to Equivalent Present Worth

n	Cash Flows in Constant Dollars	Multiplied by Discounting Factor	Equivalent Present Worth
0	-$75,000	1	-$75,000
1	30,476	$(1+0.10)^{-1}$	27,706
2	32,381	$(1+0.10)^{-2}$	26,761
3	28,334	$(1+0.10)^{-3}$	21,288
4	23,858	$(1+0.10)^{-4}$	16,295
5	45,445	$(1+0.10)^{-5}$	28,218
			$45,268

Deflation Method Example (1):
Converting actual dollars to constant dollars and then to equivalent present worth



Adjusted-Discount Method

Perform Deflation and Discounting in One Step

Step 1

$$\frac{A_n}{(1+\bar{f})^n}$$

Step 2

$$P_n = \frac{\dfrac{A_n}{(1+\bar{f})^n}}{(1+i')^n}$$

$$= \frac{A_n}{(1+\bar{f})(1+i')^n}$$

$$= \frac{A^n}{\left[(1+\bar{f})(1+i')\right]^n}$$

- Discrete Compounding

$$P_n = \frac{A_n}{(1+i)^n}$$

$$\frac{A_n}{(1+i)^n} = \frac{A_n}{\left[(1+\bar{f})(1+i')\right]^n}$$

$$(1+i) = (1+\bar{i})(1+i')$$

$$= 1 + i' + \bar{f} + i'\bar{f}$$

$$i = i' + \bar{f} + i'\bar{f}$$

- Continuous Compounding

$$i = i' + \bar{f}$$

Example (2): Adjusted-Discounted Method

Given: inflation-free interest rate = 0.10, general inflation rate = 5%, and cash flows in actual dollars

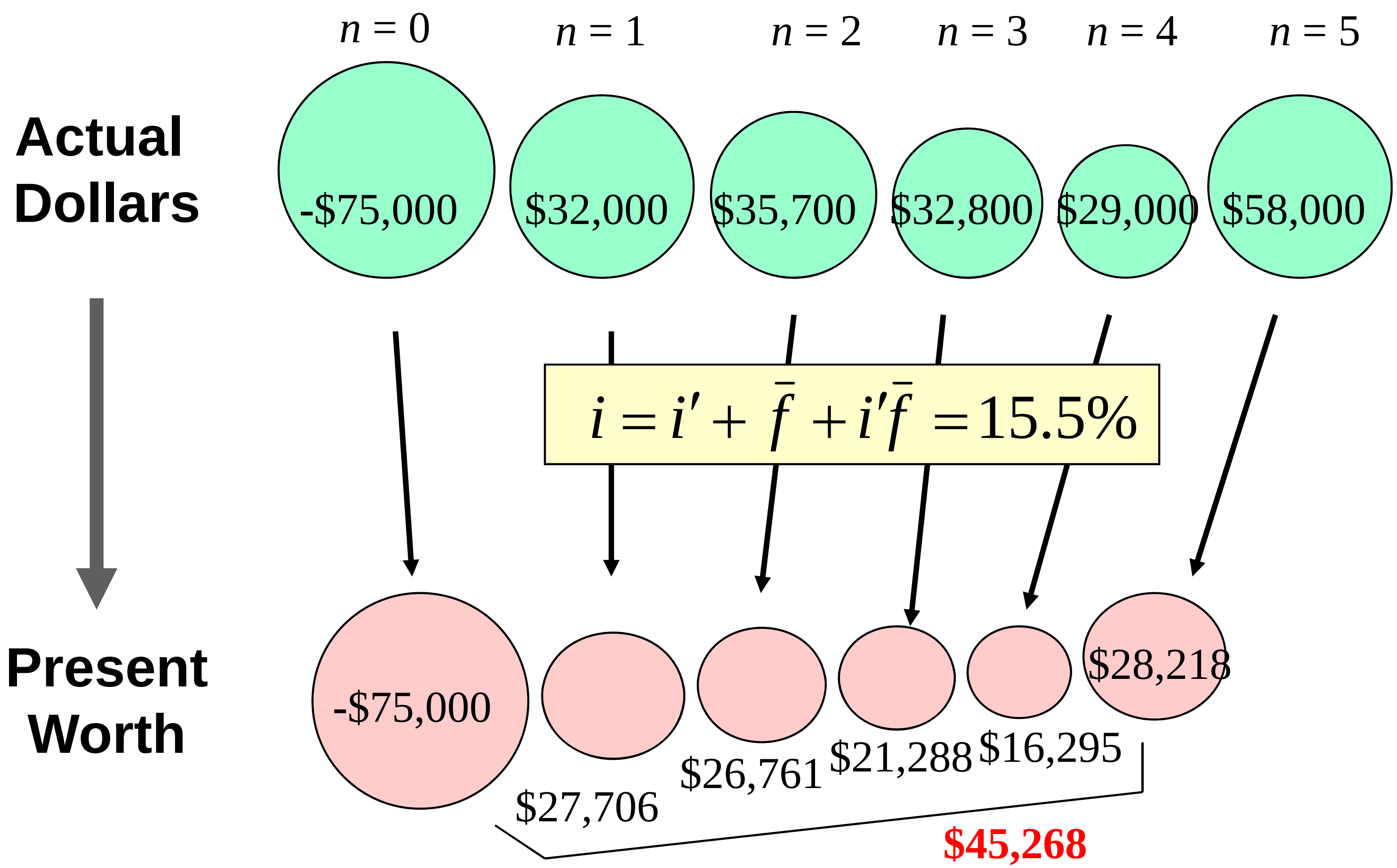
Find: *i* and NPW

$$i = i' + \bar{f} + i'\bar{f}$$
$$= 0.10 + 0.05 + (0.10)(0.05)$$
$$= 15.5\%$$

n	Cash Flows in Actual Dollars	Multiplied by	Equivalent Present Worth
0	-$75,000	1	-$75,000
1	32,000	$(1+0.155)^{-1}$	27,706
2	35,700	$(1+0.155)^{-2}$	26,761
3	32,800	$(1+0.155)^{-3}$	21,288
4	29,000	$(1+0.155)^{-4}$	16,296
5	58,000	$(1+0.155)^{-5}$	28,217
			$45,268

Adjusted Discount Method Example (2):

Converting actual dollars to present worth dollars by applying the market interest rate



Example (3): College Savings Plan
Equivalence Calculation with Composite Cash Flow Elements

Approach:

Convert any cash flow elements in constant dollars into actual dollars. Then use the market interest rate to find the equivalent present value. Assume *f* = 6% and *i* = 8% compounded quarterly.

Age (Current Age = 5 Years Old)	Estimated college expenses in today's dollars	College expenses converted into equivalent actual dollars
18 (Freshman)	$30,000	$30,000(*F*/*P*,6%,13) = $63,988
19 (Sophomore)	30,000	30,000(*F*/*P*,6%,14) = 67,827
20 (Junior)	30,000	30,000(*F*/*P*,6%,15) = 71,897
21 (senior)	30,000	30,000(*F*/*P*,6%,16) = 76,211

Solution: Required Quarterly Contributions to College Funds

$V_1 = C(F/A, 2\%, 48)$

$V_2 = \$229,211$

Let $V_1 = V_2$ and solve for C:

$C = \$2,888.48$